
Instructions

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Shuttle and Relay), at the National Final there will be a Poster Competition. All teams are required to submit a poster. The poster competition will be judged separately with a chance to win the Jacqui Lewis Trophy.

For the Poster Competition, each team will be awarded up to 6 marks which will be added to their overall score in the main competition. The mark scheme will be as follows: 6 marks for posters on the judges' shortlist, 4 marks for posters on the judges' longlist, and 2 marks where the judges find discernible effort.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Compass and straightedge constructions* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

The content of each poster is limited only by the imagination of the team members. However, on the day each team will be presented with three questions on the subject—*the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	12 marks
Imagination and presentation	12 marks
Answers to the questions	24 marks

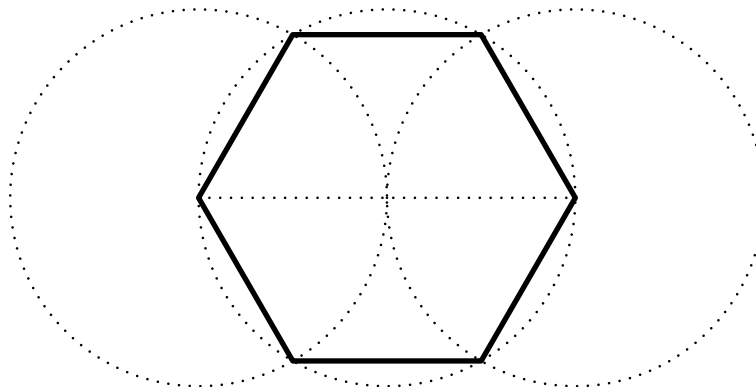
Compass and straightedge constructions

A compass and straightedge construction uses an idealised ruler and compass on a plane, as well as *Euclid's postulates*, to construct lengths, angles and points. The idealised ruler is assumed to have no markings and be infinite in length.

What are Euclid's first three postulates?

There are common constructions which you learn in mathematics classes, which form the basis of the mathematics of constructions.

A regular hexagon can be constructed by initially drawing a circle.



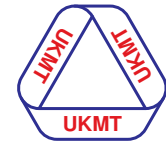
Can all regular polygons be constructed?

Given line segments having lengths a and b , it is possible to construct line segments whose lengths are the sum $a + b$ and the difference $a - b$, as well as the product $a \times b$ and their quotient a/b .

What other lengths are constructible? What angles are constructible?

It is known that, in general, it is impossible to construct the trisection of an angle.

Are there any other constructions which are impossible with a compass and straight edge?



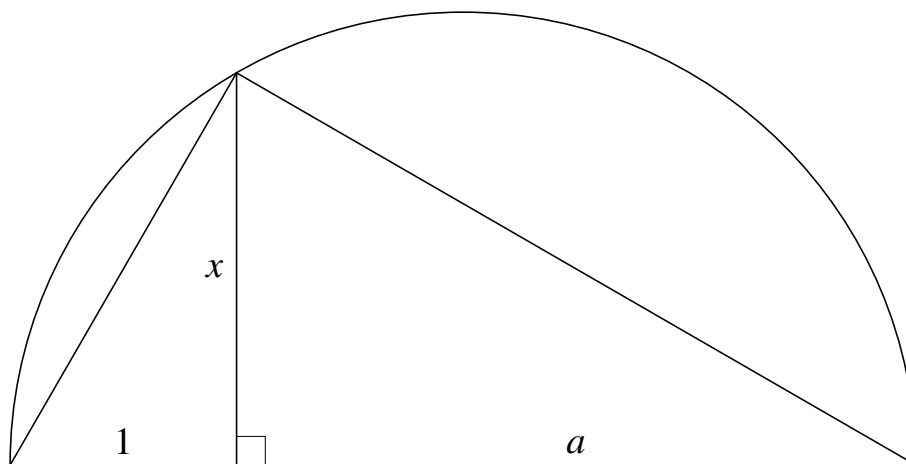
Compass and straightedge constructions

Question 1

Show how to divide a right angle into three equal angles using only compass and straightedge constructions. You should show all of your construction lines clearly in your answer.

Question 2

Given the semicircle below, prove that $x = \sqrt{a}$.



Question 3

Basic moves are defined to be constructions of the following two types:

- Type 1: Given two points A and B , draw a straight line which passes through both points;
- Type 2: Given two points O and R , draw a circle which has centre O and passes through R .

There are no other *basic moves* other than these two types.

Suppose that you are given a circle. Using no more than five basic moves, find the centre of the circle. You should show all of your construction lines clearly in your answer.

(*Example: if we are given two points A and B , we can construct an equilateral triangle which has two of its vertices at A and B . This can be done using five basic moves: two of Type 2 and three of Type 1.*)