POSTER COMPETITION

TEAM MATHS CHALLENGE
2018
NATIONAL FINAL

UKMT
United Kingdom
Mathematics Trust

Instructions

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Shuttle and Relay), at the National Final there will be a Poster Competition. All teams are required to submit a poster. The poster competition will be judged separately with a chance to win the Jacqui Lewis Trophy.

NEW FOR 2018: For the Poster Competition, each team will be awarded up to 6 marks which will be added to their overall score in the main competition. The mark scheme will be as follows: 6 marks for posters on the judges' shortlist, 4 marks for posters on the judges' longlist, and 2 marks where the judges find discernible effort.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Leonhard Euler and his Mathematics* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

> Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.
>
> A team which arrives with a poster already assembled will be disqualified.
>
> The materials of the poster must not extend beyond the edge of the A1 paper.
>
> The judges will not touch the poster, so all information must be clearly visible.
>
> Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.
>
> Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.
>
> Teams must bring with them any drawing equipment they think they will need.
>
> Glue sticks and scissors will be provided.

The content of each poster is limited only by the imagination of the team members. However, on the day each team will be presented with three questions on the subject—*the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	12 marks
Imagination and presentation	12 marks
Answers to the questions	24 marks

POSTER
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TOPIC

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Leonhard Euler and his Mathematics

Leonhard Euler (1707–1783) was born in Basel, Switzerland. In his career as a mathematician, he studied topics including number theory, combinatorics, geometry, mathematical analysis, and the stability of ships, as well as being considered the father of graph theory.

With a life as mathematically rich as Euler's, there are many potential areas of his work you could focus on. Here are some ideas that should be included in your research.

A *graph* is a mathematical structure that consists of vertices and edges. It can be used as a simpler way to represent complex problems in mathematics, such as the famous *Seven Bridges of Königsberg* problem. A *planar graph* is a graph drawn in the plane in which no edge crosses another edge. Below is a planar graph consisting of four vertices, six edges and four faces, where each vertex has degree three.
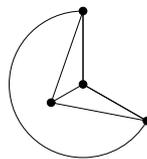
> Can all graphs be drawn as planar graphs?

A *polyhedron* is a three-dimensional shape with v vertices, e edges and f plane faces. Euler discovered that for all polyhedra $v + f - e = 2$. This is known as Euler's formula.

> Are there results analogous to Euler's formula in other dimensions?

A famous problem studied by Euler is the *Thirty-Six Officers Problem*. It asks whether it is possible to arrange six regiments, each containing six officers, each of a different rank, in a 6×6 square so that no rank or regiment is repeated in any row or column.

> Is there a solution to the Thirty-Six Officers Problem?

Through Euler's work on combinatorics, he began working on what would later be named *Catalan numbers*.

> In how many ways can a regular $(n + 2)$-sided polygon
> be divided into n triangles?

POSTER
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QUESTIONS

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Leonhard Euler and his Mathematics

Question 1

Draw a planar graph with six vertices and nine edges. Each vertex should be of degree three.

Question 2

Show that it is possible to arrange four regiments $\{A, B, C, D\}$, each consisting of four officers, each of a different rank $\{1, 2, 3, 4\}$, in a 4×4 grid, such that no regiment or rank is repeated in any row or column.

Here is a solution for three regiments, each consisting of three officers, each of a different rank.

$C2$	$A3$	$B1$
$B3$	$C1$	$A2$
$A1$	$B2$	$C3$

Question 3

Buckminsterfullerene is a form of carbon with chemical formula C_{60}. Its molecules, often known as *buckyballs*, are made of sixty carbon atoms each. The atoms in a buckyball form a *polyhedron*, with an atom at each vertex. Each vertex of the polyhedron is surrounded by exactly two regular hexagons and exactly one regular pentagon and no other polygons.

   (a) How many edges and faces does the polyhedron have?

   (b) How many of the faces of the polyhedron are hexagons?

You must show all your working for each part.