Instructions

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Shuttle and Relay), at the National Final there will be a Poster Competition, with a chance to win the Jacqui Lewis Trophy.

All teams are required to submit a poster. The poster competition will be judged separately and will not affect the Team Maths Challenge score, but forms an integral part of the National Final.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Folding* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

The content of each poster is limited only by the imagination of the team members. However, on the day each team will be presented with three questions on the subject—*the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	12 marks
Imagination and presentation	12 marks
Answers to the questions	24 marks

Folding

When paper-folding is mentioned, your thoughts may immediately turn to origami and folding objects like birds or flowers. This artistic side of paper-folding is well-known, but it is less well known that there are mathematical aspects too, and many of them. For example, it is possible to fold some simple mathematical shapes. Note that, here and elsewhere, we use the expression 'fold [something]' to mean 'create [something] by folding a piece of paper along straight lines'.

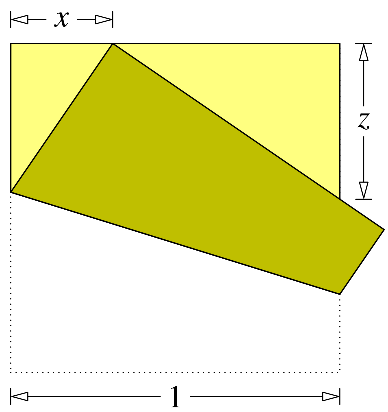
> Starting from a rectangular (or square) piece of paper
> what simple mathematical shapes is it possible to fold?

One natural question to ask is "what lengths can be folded?"

> What is Haga's theorem?

Start with a square sheet of paper with sides of length 1.

Given some length $x < 1$, show how to calculate the value of z in the diagram alongside in terms of x.



> Can *any* rational length (less than one) be folded?

Anyone familiar with classical 'straightedge and compass' constructions may be surprised to learn that everything that can be constructed in the classical way may also be achieved by folding paper, indeed, even more is possible by paper-folding than by classical constructions. For example, a given angle may be trisected, something that is not possible with straightedge and compass.

> What constructions are possible by folding
> but impossible using 'straightedge and compass'?

Folding

Question 1

A square sheet of paper is folded *once* along a straight line.

For each of the following shapes, either show how to make it, or prove that it cannot be made:

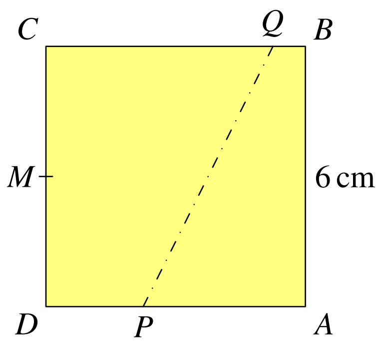
(a) a rectangle that is not a square;
(b) an isosceles triangle;
(c) a right-angled triangle;
(d) a rhombus that is not a square.

Question 2

The diagram shows a square sheet of paper $ABCD$ with sides of length 6 cm. The point M is the midpoint of side CD.

The paper is folded along PQ, where P lies on DA and Q lies on BC, so that the vertex A folds to the point M.

What is the length of PQ?



Question 3

The square sheet of paper S has sides of length 2.

Explain how to fold S along straight lines to make a line segment of length $\sqrt{3}$.