Instructions

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Shuttle and Relay), at the National Final there will be a Poster Competition, with a chance to win the Jacqui Lewis Trophy.

All teams are required to submit a poster. The poster competition will be judged separately and will not affect the Team Maths Challenge score, but forms an integral part of the National Final.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Colouring* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

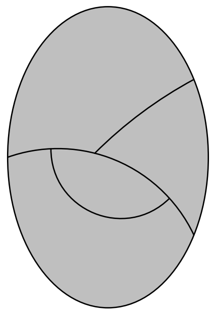
The content of each poster is limited only by the imagination of the team members. However, on the day each team will be presented with three questions on the subject—*the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	12 marks
Imagination and presentation	12 marks
Answers to the questions	24 marks

Colouring

Given a map showing various regions, a natural question to ask is: can we colour the map? We assume that each region is given a single colour, and that two regions which have a boundary in common have different colours. Two regions that meet in a single point are not considered to have a boundary in common.



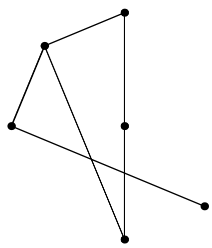
It is clear that any map may be coloured in this way—simply choose a different colour for each region—but usually we want to achieve a colouring using the smallest possible number of colours. Finding the minimum number of colours required is not always easy!

> What is the *Four Colour Theorem*?
> Why was the original proof of the Four Colour Theorem controversial?

> What is the *Five Colour Theorem*?

> What happens when the map is not drawn on a plane but instead is drawn on some other surface?

Rather than colouring a map, in mathematics it is more common to consider colouring a *graph*. In this context, a graph is a collection of points with lines connecting some of them, such as the example shown alongside. The points are called the *nodes* of the graph, and the lines are the *edges*. The graph shown has six nodes and six edges.



A map may be converted to a graph by creating a node for each region and, for each boundary between two regions, joining the two corresponding nodes by an edge.

There are two standard methods of colouring a graph. In a *vertex colouring*, each node is coloured so that no edge joins identically coloured nodes; in an *edge colouring*, each edge is coloured so that edges meeting at the same vertex have different colours.

> What is the *Petersen graph*?

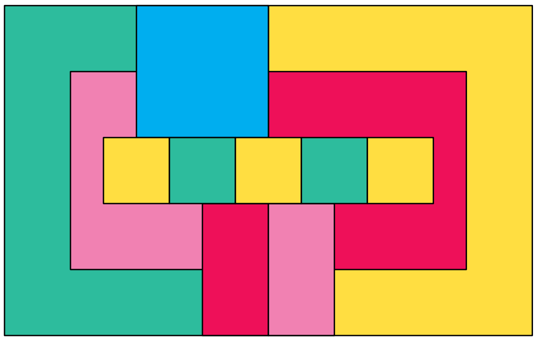
> What can be said about vertex colourings of the Petersen graph?
> What can be said about edge colourings of the Petersen graph?

Colouring

Question 1

The diagram shows a “map” with twelve regions. Each region is coloured in one of five colours so that regions of the same colour do not meet.

Show how to colour the map with just four colours so that regions of the same colour do not meet.

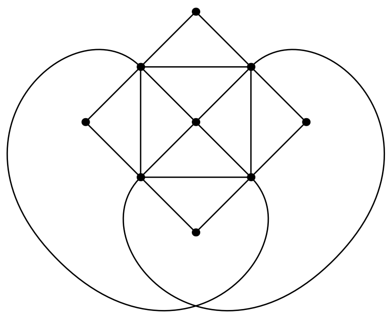


Question 2

The diagram shows a graph G with nine nodes and eighteen edges; two of the edges are shown as curves.

(a) What is the smallest number of colours needed in a *vertex colouring* of G?

(b) Prove that your answer is the smallest.



Question 3

The diagram shows a graph H with sixteen nodes and twenty-four edges.

(a) What is the smallest number of colours needed in an *edge colouring* of H?

(b) Prove that your answer is the smallest.