UKMT Team Maths Challenge

National Final

Monday 17th June 2013

Poster Round

Instructions

In addition to four rounds similar to those at the regional finals (Group Circus, Crossnumber, Head-to-Head and Relay), there will be a Poster Competition at the National Final. All teams are required to submit a poster. This will be judged separately and will not affect the Team Maths Challenge score, but forms an integral part of the National Final.

After the competition some posters may be retained by the UKMT in order to be reproduced for promotional purposes.

On the day, teams will have 50 minutes to create a poster on a sheet of A1 paper (landscape), which will be provided. Sheets of A4 paper will also be available.

The subject of the poster will be *Packing* (see overleaf). Teams must carry out research into this topic in the weeks leading up to the final.

Teams may create materials beforehand, but such prepared work must be on sheets no larger than A4 and must be assembled to create the poster on the day.

A team which arrives with a poster already assembled will be disqualified.

The materials of the poster must not extend beyond the edge of the A1 paper.

The judges will not touch the poster, so all information must be clearly visible.

Your team number (assigned to you on arrival) must be clearly visible in the bottom right-hand corner of the poster. There must be nothing else on the poster to identify the team.

Reference books may not be used at the competition, and large extracts copied directly from books or the internet will not receive much credit.

Teams must bring with them any drawing equipment they think they will need.

Glue sticks and scissors will be provided.

The content of each poster is limited only by the imagination of the team members. However, on the day each team will be presented with three questions on the subject—*the answers to these questions must be incorporated into the structure of the poster*. Teams may be asked to provide geometric or algebraic proofs, and some ingenuity may be involved.

Posters will be judged on the following criteria:

General mathematical content	12 marks
Imagination and presentation	12 marks
Answers to the questions	24 marks

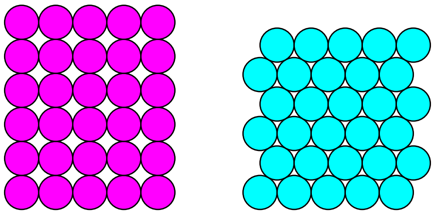
Packing

A *packing* is the arrangement of copies of a given shape, without overlap, in a specified region. Usually the shapes to be packed are all of equal size, but they need not be. Examples include *circle packing*, *square packing* and *triangle packing*.

What other types of packing are there?

Two examples are the so-called *square* and *hexagonal* packings of equal circles in the plane. Joining the centres of touching circles gives plane tessellations by a square and by an equilateral triangle.

The fraction of the area of the region occupied by the packed shapes is the *density* of a packing.



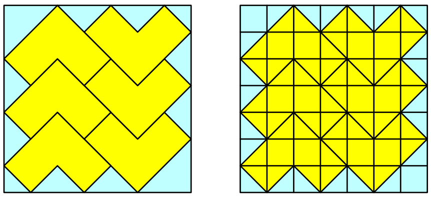
From the figures, it seems clear that the density of the hexagonal packing is greater than that of the square packing. Can you prove this?

What is *Kepler's conjecture*?

Answering the natural question "what is the greatest possible density?" may be difficult when the region to be packed is finite. Even simple cases remain unsolved and there have been some well-known incorrect conjectures.

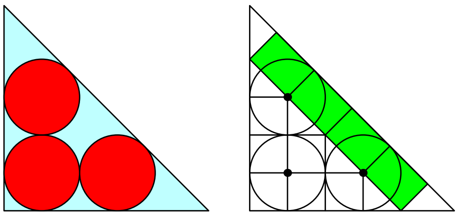
What is *Malfatti's problem*?

An L-triomino is made by joining together three 1×1 squares. The first figure on the right shows a packing of six L-triominoes into a square. One way to find the density is to dissect the figure into small squares and half-squares, as shown.



What is the density of this packing?

The first figure shows three circles of radius 1 packed into an isosceles right-angled triangle. To find the area, dissect into 1×1 squares, half-squares, and rectangles, which measure $\sqrt{2} \times 1$ from Pythagoras' Theorem. To start the dissection, join the centres of the circles, and join each centre to the points of tangency.



What is the density of this packing?

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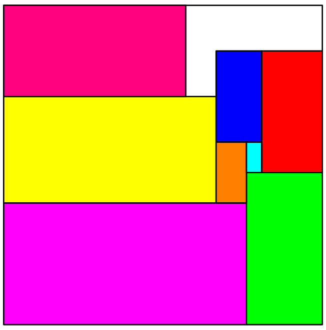
Packing

Question 1

A *domino* consists of two squares joined together *edge to edge*. The diagram shows eight different coloured dominoes packed into a square.

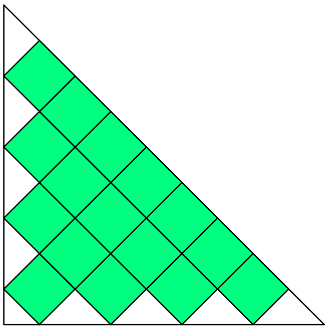


The dominoes are of size $n \times 2n$ for $n = 1$ to 8, that is, 1×2, 2×4, 3×6, $\ldots$, 8×16.

What is the density of the packing? In other words, what fraction of the area of the square is occupied by the dominoes?

Question 2

A unit square has sides of length 1. The figure shows 16 unit squares packed into an isosceles right-angled triangle.



Prove that there is a packing with density

$$\left(\frac{2n}{2n+1}\right)^2$$

of n^2 unit squares into an isosceles right-angled triangle, where n is any positive integer.

Question 3

Let T_n be the nth triangular number, so $T_1 = 1$, $T_2 = 3$, $T_3 = 6$, $\ldots$.

Prove that T_n circles with radius 1 may be packed into an isosceles right-angled triangle with area $2n^2 + 1 + 2n\sqrt{2}$.