United Kingdom Mathematics Trust

Maths Challenges News

Issue 45 June 2014

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Mathematical Challenges

The UKMT's activities are wide and varied, but its focal point is the organisation of national mathematics competitions, from the popular mass challenges and team competitions, to the selection and training of the UK team for international competitions including the prestigious International Mathematical Olympiad.

National mathematics competitions have existed in the UK for several decades, and throughout its 18-year existence, the UKMT has continued to nurture and expand the number of competitions. As a result, almost 650,000 students across the country participate in the UKMT Mathematical Challenges alone, and their teachers provide much valued encouragement and help to support participation.

Our recent initiatives have seen the development of new competitions such as the Senior Kangaroo, and Mathematical Olympiad for Girls (see the article overleaf), and we are planning to launch another new follow-on competition, the Junior Kangaroo, in June 2015. We now publish additional material on our website after each of our challenges; have you seen the "Solutions and Investigations"? These solutions augment the printed solutions sent to schools, and some additional exercises are included for further investigation. Teachers also tell us how much their students value their certificates, so last year we introduced new Best in School Year certificates, and for this year's JMC, we produced a downloadable participation certificate which can be awarded to all participants.

As the focus on problem solving becomes more important in the classroom, the UKMT Maths Challenges are ideal for testing your students' skills in this area. Do consider entering all your students for the challenges, as they can sometimes throw up unexpected results! Entries for the 2014/15 challenges will open in July, so keep your eye out for the launch. We'll send an email to let you know when entries are being taken, and let you know via twitter, so please follow us @UKMathsTrust. Entry forms will be posted early in the new academic year.

SMC 2013 Q2.

Little John claims he is 2m 8cm and 3mm tall. What is this height in metres?

A 2.83m B 2.803m C 2.083m D 2.0803m E 2.0083m

IMC 2014 Q14.

This year the *Tour de France* starts in Leeds on 5 July. Last year, the total length of the *Tour* was 3404 km and the winner, Chris Froome, took a total time of 83 hours 56 minutes 40 seconds to cover this distance. Which of these is closest to his average speed over the whole event?

A 32km/h B 40 km/h C 48 km/h D 56 km/h E 64 km/h

JMC 2014 Q5.

What is the difference between the smallest 4-digit number and the largest 3-digit number?

A 1 B 10 C 100 D 1000 E 9899

UKMT Contact Details

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Diary Dates for 2014-15

Challenge	*Date*	*Follow-on Round*	*Date*
Senior	Thursday 6 November 2014	BMO1 and Senior Kangaroo BMO2	Friday 28 November 2014 Thursday 29 January 2015
Intermediate	Thursday 5 February 2015	IMOK (Olympiad and Kangaroo)	Thursday 19 March 2015
Junior	Thursday 30 April 2015	JMO (Olympiad and new Kangaroo)	Tuesday 9 June 2015

Entry Details for Mathematical Olympiad for Girls

Do you have talented and committed female mathematicians who are looking for a new challenge? Then enter them for the 2014 UK Mathematical Olympiad for Girls, taking place on Tuesday 23rd September.

But what is the Mathematical Olympiad for Girls? Affectionately known as MOG, this new Olympiad was added to the list of competitions run by the UK Mathematics Trust in 2011. MOG's aim is to encourage enthusiastic girls to develop their passion for mathematics, particularly at Olympiad level. MOG continues to go from strength to strength, from small beginnings in 2011 which saw only 155 entries, to over 1,200 entries from female mathematicians throughout the UK last year.

MOG also forms part of the process to identify potential squad members for the annual European Girls' Mathematical Olympiad (the 2015 event is to be held in Minsk, Belarus) and other international competitions. Therefore, although age does not matter, students should be eligible to be part of the UK team (either eligible for a UK passport describing them as a British Citizen, or will have completed 3 full years of full-time secondary education in the UK by the time they leave school).

In 2014 MOG will be held on **Tuesday 23rd September.** To register your student for MOG2014, please email us at **enquiry@ukmt.org.uk** with the following information: student full name, date of birth, school year, school name, UKMT centre number, and confirmation of student eligibility. Your registration will be acknowledged and the paper will be sent *by email* to the contact teacher on Monday 22nd September for the paper to be taken on Tuesday 23rd September**.** Entry to the competition is free of charge.

Students need to be confident mathematicians, and will probably already have achieved some success at UKMT follow-on rounds or in the mentoring schemes. To give some indication of the type of questions involved, MOG past papers can be found at **www.bmoc.maths.org/home/egmo** and we encourage all interested students to attempt some of these before entering the competition, as Olympiad questions may be different in style to anything they have previously attempted. The paper contains five questions to be answered in two-and-a-half hours.

Try Question 2 from MOG 2013:

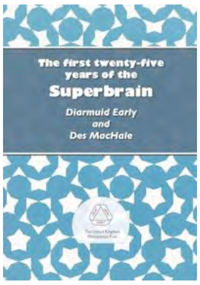
In triangle ABC, the median from A is the line AM, where M is the midpoint of the side BC. In any triangle, the three medians intersect at the point called the centroid, which divides each median in the ratio 2 : 1.

In the convex quadrilateral $ABCD$, the points A', B', C' and D' are the centroids of the triangles BCD, CDA, DAB and ABC, respectively.

a) By considering the triangle MCD, where M is the midpoint of AB, prove that $C'D'$ is parallel to DC and that $C'D' = 1/3\,DC$.

b) Prove that the quadrilaterals $ABCD$ and $A'B'C'D'$ are similar.

Publications Snippets



"The first twenty-five years of the Superbrain" by Diarmuid Early and Des MacHale is an extraordinary collection of mathematical problems, laced with some puzzles. Superbrain is a competition based at the University of Cork, and has developed a following all over Ireland, both in schools and universities. People who enjoy mathematical Olympiad problems will certainly enjoy Superbrain, with algebra, combinatorics, geometry and number theory well represented. There are some very entertaining and witty questions accessible to many, and also some ingenious problems which require more serious analysis.

To find out more about this and our extensive range of publications, and to order online, see our website at **www.publications.ukmt.org.uk**.

The Power of Double Counting

When a student first comes across the formula $1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$, for each positive integer n, it ought to come as a shock. There are predictable ways to prove it by induction, and a nice geometric argument involving one square of side 1, two squares of side 2, and so on up to n squares of side n, which fit together (with balancing overlaps and holes) to form a square of side $1 + 2 + \dots + n$.

This note is a challenge to find a double counting argument which establishes the formula. A double counting argument arises when you count something correctly in two different ways, and then put an equals sign between your answers. There is a quick double counting proof of the *handshaking lemma*, in which you count “hands involved in handshakes” in two different ways.

In the case of our formula, imagine that you have an $n \times n$ version of a chessboard. Define a sub-board to be a non-trivial rectangle made of the small squares of the board. Now, if $n = 2$, there are four sub-boards which are 1×1 squares, two sub-boards which are horizontal 1×2 dominos, two sub-boards which are vertical 2×1 dominos, and finally one sub-board which is the whole 2×2 board itself. Thus there are $4 + 2 + 2 + 1 = 9$ sub-boards.

Notice that $(1 + 2)^2 = 9 = 1^3 + 2^3$. The task is to find two different ways of counting the number of sub-boards of an $n \times n$ chessboard, and deduce the identity. If you find yourself doing a lot of onerous algebra, then step back and start again. There is a better way!

Geoff Smith @GeoffBath, University of Bath

Processing the JMC

We were thrilled to receive over 290,000 entries to this year’s Junior Mathematical Challenge, but what happens once you send them back to UKMT? Our staff in the packing office carefully unpack your scripts and process them, one school at a time, through the Optical Mark Readers (OMR), ensuring all scripts are read by the machine. If a pupil does not correctly complete the top of their answer sheet (common errors include using circles instead of lines, or doodling down the side), the sheet is rejected by the machine and it is up to our staff to spot the error and correct it using eraser and pencil, before running it once again through the machine. We never amend answers! We process around 25,000 sheets each day, so how long did it take us to process the JMC? Once all the sheets have been processed, we check the data and set the thresholds before emailing the results to our school contact. Once results have been sent electronically, they are printed and sent through the post.

Mathematical Circles are coming round again!

Don’t miss the chance for your pupils to take part in a Mathematical Circle if your school receives an invitation this year.

The UKMT Mathematical Circles are two day non-residential events for pupils who are fifteen years old or thereabouts. We aim to recruit about forty pupils to each of these events by inviting local schools to each nominate two or three pupils. At each Mathematical Circle the pupils work in groups on a variety of topics, with the aims of enriching their experience of mathematics, inspiring them to continue their study of mathematics beyond school, and enabling them to meet like-minded pupils from other schools in their area.

Hull Mathematical Circle

The response of pupils to our Mathematical Circles has been very enthusiastic. Here are a few quotes from last year:

“I had always thought about studying maths further and now I am certain I will”, “Great fun, thanks!”, “An excellent opportunity to learn something unique!”, “Good to mix with other schools”, “My brain hurts, but in a good way!”, “I thought it was great – it was fun because it took a lot more thinking than school maths”.

Cumbria Mathematical Circle

The first UKMT Mathematical Circle was held at Hutchesons’ Grammar School in Glasgow in 2012. With the help of a grant from the DfE, the programme has now been expanded. We ran twelve Mathematical Circles over the past year, and we are planning to run sixteen next year. Unfortunately, we still don’t have the resources to run Mathematical Circles in every part of the UK. If we hold one in your area, and you receive an invitation to nominate pupils to take part, think yourself lucky, and please take advantage of the opportunity!

Balkans Mathematical Olympiad

Congratulations to the team representing the UK at the Balkans Mathematical Olympiad, held in Pleven, Bulgaria, in May. The UK has a self-imposed rule that no student may participate in this competition more than once, ensuring more students gain international experience. The team of Joe Benton (St Paul's School), Liam Hughes (Robert Smyth Academy), Neel Nanda (Latymer School), Linden Ralph (Hills Road VI Form College), Kasia Warburton (Reigate Grammar School) and Harvey Yau (Ysgol Dyffryn Taf) put in a great performance, receiving one silver medal, two bronze medals, and three honourable mentions for fully solving one problem. Thanks to Team Leader Jack Shotton (Imperial College London) and Deputy Leader Dr Gerry Leversha (formerly of St Paul's School). Further details about the event can be found at **www.bmoc.maths.org**.

Try problem four from the Balkans Mathematical Olympiad. This problem was proposed by the UK (Sahl Khan) :

Let n be a positive integer. A regular hexagon with side length n is divided into equilateral triangles with side length 1 by lines parallel to its sides.

Find the number of regular hexagons all of whose vertices are among the vertices of the equilateral triangles.

Senior Team Maths Challenge

In February, the winning teams from the 55 regional heats competed in the 2013/14 Senior Team Mathematics Challenge (STMC) Final at the Camden Centre, London. 1,146 schools and colleges from around the UK participated in the STMC. Many congratulations to Hampton School (pictured), overall winners of the Senior Team Maths Challenge 2013/14, and to The Grammar School at Leeds who won the poster competition.

Entry forms for next year's competition, which is organised jointly by the UKMT and Further Mathematics Support Programme (FMSP), will be arriving in schools shortly along with a copy of the winning poster from the National Final. The 2013/14 material can be downloaded from **www.stmc.ukmt.org.uk**.

Prize Sudoku

Medium – warm up on this one!

			6	4			8	
	9			1		3		6
	5			2		4		
	8				7			2
5	6						1	9
3			8				5	
		8		9			2	
6		2		7			4	
	3			8	2			

In Sudoku, every digit from 1 to 9 must appear in each of the nine rows, each of the nine columns, and each of the nine outlined boxes.

A draw from the correct entries will take place after the closing date and the winner will receive a book with a mathematical theme.

PRIZE puzzle - a little harder! →

6	3	7	8		1			9
					4		8	
4	8							
8	6				3			5
			2	5	8			
2			7				4	8
							1	6
	9		6					
5			3		9	8	2	4

Please send entries (photocopies accepted) by the closing date of Friday 18 July 2014 to:
Sudoku, UKMT, School of Maths Satellite, University of Leeds, Leeds LS2 9JT

NAME……………………………………SCHOOL ADDRESS ....

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