



**United Kingdom  
Mathematics Trust**

## **Mentoring Scheme**

Supported by **Ox**FORD  
ASSET MANAGEMENT

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### **Sample questions**

These questions are taken from the first sheet and give a good indication of the level of difficulty and prerequisite knowledge required at the start of the programme.

1. Does there exist a square number whose digit sum is 123? Justify your answer.
2. The collection  $\{8, 9, 12\}$  is a set of three positive integers with the property that, given any two of these integers, their greatest common divisor (gcd) is equal to their difference:
  - $\text{gcd}(8, 9) = 1 = 9 - 8$ ;
  - $\text{gcd}(8, 12) = 4 = 12 - 8$ ;
  - $\text{gcd}(9, 12) = 3 = 12 - 9$ .

Can you find a set of four integers with the same property? What about five? Or six?

3. The point  $P$  lies on the circumcircle of the triangle  $ABC$ . Perpendiculars  $PL$ ,  $PM$  and  $PN$  are drawn to the sides  $BC$ ,  $CA$  and  $AB$  respectively. If necessary, the sides of the triangle are produced (extended). Prove that the points  $L$ ,  $M$  and  $N$  are collinear (lie on the same straight line).