

United Kingdom
Mathematics Trust

INTERMEDIATE MATHEMATICAL OLYMPIAD

MACLAURIN PAPER

Thursday 21 March 2019

Organised by the United Kingdom Mathematics Trust



England & Wales: Year 11

Scotland: S4

Northern Ireland: Year 12

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π , fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

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www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
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- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do not hand in rough work.*

1. A train leaves K for L at 09:30 while another train leaves L for K at 10:00. The first train arrives in L 40 minutes after the trains pass each other. The second train arrives in K 1 hour and 40 minutes after the trains pass.

Each train travels at a constant speed.

At what time did the trains pass each other?

2. A right-angled triangle has area 150 cm^2 and the length of its perimeter is 60 cm.

What are the lengths of its sides?

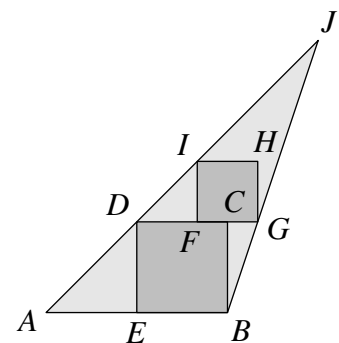
3. Two numbers are such that the sum of their reciprocals is equal to 1. Each of these numbers is then reduced by 1 to give two new numbers.

Prove that these two new numbers are reciprocals of each other.

[The reciprocal of a non-zero number x is the number $\frac{1}{x}$.]

4. The diagram shows the two squares $BCDE$ and $FGHI$ inside the triangle ABJ , where E is the midpoint of AB and C is the midpoint of FG .

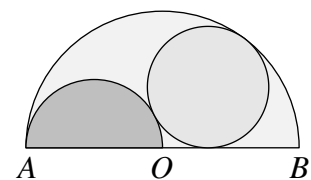
What is the ratio of the area of the square $BCDE$ to the area of the triangle ABJ ?



5. A semicircle of radius 1 is drawn inside a semicircle of radius 2, as shown in the diagram, where $OA = OB = 2$.

A circle is drawn so that it touches each of the semicircles and their common diameter, as shown.

What is the radius of the circle?



6. A tiling of an $n \times n$ square grid is formed using 4×1 tiles.

What are the possible values of n ?

[A tiling has no gaps or overlaps, and no tile goes outside the region being tiled.]