The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Maclaurin Paper

Thursday 16th March 2017

All candidates must be in *School Year 11* (England and Wales), *S4* (Scotland), or *School Year 12* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.

   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

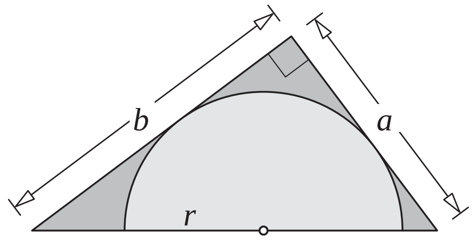
   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.

   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

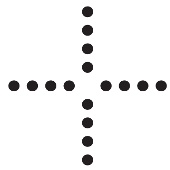
M1. The diagram shows a semicircle of radius r inside a right-angled triangle. The shorter edges of the triangle are tangents to the semicircle, and have lengths a and b. The diameter of the semicircle lies on the hypotenuse of the triangle.



Prove that

$$\frac{1}{r} = \frac{1}{a} + \frac{1}{b}.$$

M2. How many triangles (with non-zero area) are there with each of the three vertices at one of the dots in the diagram?



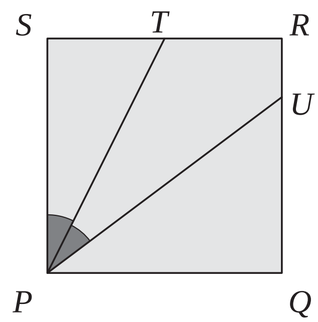
M3. How many solutions are there to the equation

$$m^4 + 8n^2 + 425 = n^4 + 42m^2,$$

where m and n are integers?

M4. The diagram shows a square $PQRS$ with sides of length 2. The point T is the midpoint of RS, and U lies on QR so that $\angle SPT = \angle TPU$.

What is the length of UR?



M5. Solve the pair of simultaneous equations

$$(a + b)(a^2 - b^2) = 4 \qquad \text{and}$$

$$(a - b)(a^2 + b^2) = \frac{5}{2}.$$

M6. The diagram shows a 10×9 board with seven 2×1 tiles already in place.
What is the largest number of additional 2×1 tiles that can be placed on the board, so that each tile covers exactly two 1×1 cells of the board, and no tiles overlap?