

United Kingdom  
Mathematics Trust

# INTERMEDIATE MATHEMATICAL OLYMPIAD

## MACLAURIN PAPER

Thursday 19 March 2020

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supported by  

*England & Wales: Year 11*

*Scotland: S4*

*Northern Ireland: Year 12*

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

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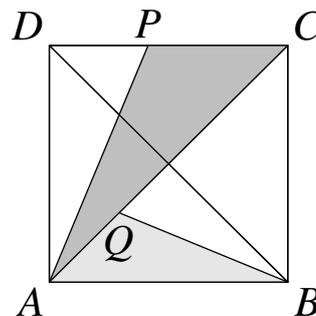
- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
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- ◇ *Do not hand in rough work.*

1. A bag contains counters, of which ten are coloured blue and  $Y$  are coloured yellow. Two yellow counters and some more blue counters are then added to the bag. The proportion of yellow counters in the bag remains unchanged before and after the additional counters are placed into the bag.

Find all possible values of  $Y$ .

2. In the square  $ABCD$ , the bisector of  $\angle CAD$  meets  $CD$  at  $P$  and the bisector of  $\angle ABD$  meets  $AC$  at  $Q$ .

What is the ratio of the area of triangle  $ACP$  to the area of triangle  $BQA$ ?

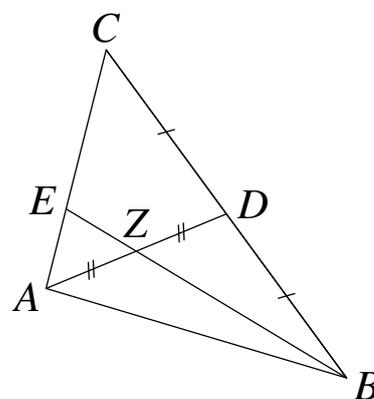


3. An altitude of a triangle is the shortest distance from a vertex to the line containing the opposite side.

Find the side lengths of all possible right-angled triangles with perimeter 5 cm and shortest altitude 1 cm.

4. The diagram shows a triangle  $ABC$  and two lines  $AD$  and  $BE$ , where  $D$  is the midpoint of  $BC$  and  $E$  lies on  $CA$ . The lines  $AD$  and  $BE$  meet at  $Z$ , the midpoint of  $AD$ .

What is the ratio of the length  $CE$  to the length  $EA$ ?



5. Let  $p$  and  $q$  respectively be the smallest and largest prime factors of  $n$ . Find all positive integers  $n$  such that  $p^2 + q^2 = n + 9$ .

6. Seth and Cain play a game. They take turns, and on each turn a player chooses a pair of integers from 1 to 50. One integer in the pair must be twice the other, and the players cannot choose any integers used previously. The first player who is unable to choose such a pair loses the game. If Seth starts, determine which player, if any, has a winning strategy.