UK Junior Mathematical Olympiad 2015

Organised by The United Kingdom Mathematics Trust

Tuesday 9th June 2015

RULES AND GUIDELINES : READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, measuring instruments and squared paper is forbidden.**
3. All candidates must be in *School Year 8 or below* (England and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work. Write in blue or black pen or pencil.

   For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

   Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

   Do not hand in rough work.
5. Questions A1-A10 are relatively short questions. Try to complete Section A within the first 30 minutes so as to allow well over an hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*.
   This means that each answer must be accompanied by clear explanations and proofs.
   Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be challenging! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you are not able to do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Answers must be FULLY SIMPLIFIED, and EXACT using symbols like π, fractions, or square roots if appropriate, but NOT decimal approximations.

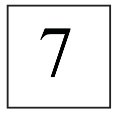
DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

Try to complete Section A within 30 minutes or so. Only answers are required.

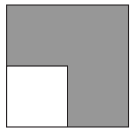
A1. It is 225 minutes until midnight. What time is it on a 24-hour digital clock?

A2. The diagram shows what I see when I look straight down on the top face of a non-standard cubical die. A positive integer is written on each face of the die. The numbers on every pair of opposite faces add up to 10. What is the sum of the numbers on the faces I cannot see?



A3. The diagram shows one square inside another. The perimeter of the shaded region has length 24 cm.

What is the area of the larger square?

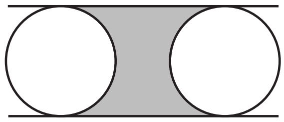


A4. My fruit basket contains apples and oranges. The ratio of apples to oranges in the basket is 3 : 8. When I remove one apple the ratio changes to 1 : 3.

How many oranges are in the basket?

A5. Two circles of radius 1 cm fit exactly between two parallel lines, as shown in the diagram. The centres of the circles are 3 cm apart.

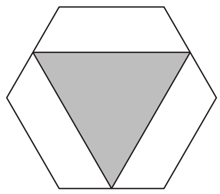
What is the area of the shaded region bounded by the circles and the lines?



A6. There are 81 players taking part in a knock-out quiz tournament. Each match in the tournament involves 3 players and only the winner of the match remains in the tournament − the other two players are knocked out. How many matches are required until there is an overall winner?

A7. The diagram shows an equilateral triangle inside a regular hexagon that has sides of length 14 cm. The vertices of the triangle are midpoints of sides of the hexagon.

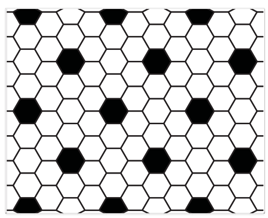
What is the length of the perimeter of the triangle?



A8. What is the units digit in the answer to the sum $9^{2015} + 9^{2016}$?

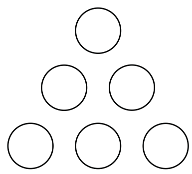
A9. The figure shows part of a tiling, which extends indefinitely in every direction across the whole plane. Each tile is a regular hexagon. Some of the tiles are white, the others are black.

What fraction of the plane is black?



A10. Lucy wants to put the numbers 2, 3, 4, 5, 6 and 10 into the circles so that the products of the three numbers along each edge are the same, and as large as possible.

What is this product?



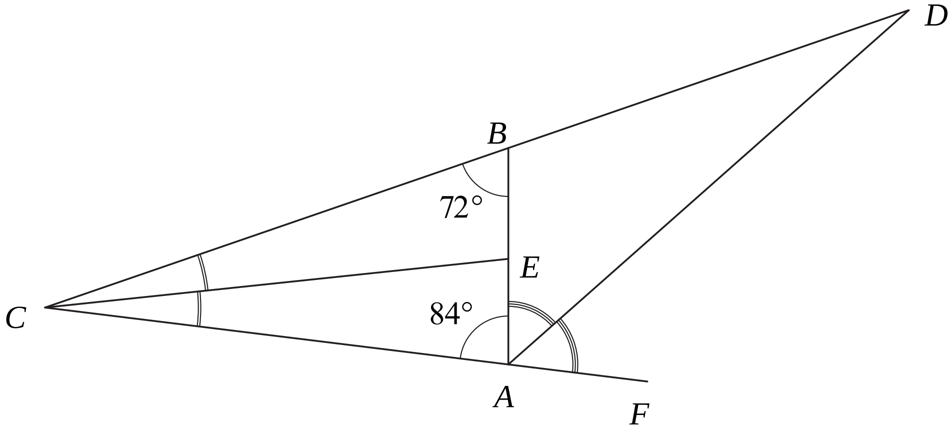
Section B

Your solutions to Section B will have a major effect on your JMO result. Concentrate on one or two questions first and then **write out full solutions** (not just brief 'answers').

B1. Let N be the smallest positive integer whose digits add up to 2015.

What is the sum of the digits of $N + 1$?

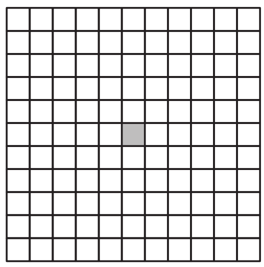
B2. The diagram shows triangle ABC, in which $\angle ABC = 72°$ and $\angle CAB = 84°$. The point E lies on AB so that EC bisects $\angle BCA$. The point F lies on CA extended. The point D lies on CB extended so that DA bisects $\angle BAF$.



Prove that $AD = CE$.

B3. Jack starts in the small square shown shaded on the grid, and makes a sequence of moves. Each move is to a neighbouring small square, where two small squares are neighbouring if they have an edge in common. He may visit a square more than once.

Jack makes four moves. In how many different small squares could Jack finish?



B4. The point F lies inside the regular pentagon $ABCDE$ so that $ABFE$ is a rhombus. Prove that EFC is a straight line.

B5. I have two types of square tile. One type has a side length of 1 cm and the other has a side length of 2 cm.

What is the smallest square that can be made with equal numbers of each type of tile?

B6. The letters a, b, c, d, e and f represent single digits and each letter represents a different digit. They satisfy the following equations:

$$a + b = d, \qquad b + c = e \qquad \text{and} \qquad d + e = f.$$

Find all possible solutions for the values of a, b, c, d, e and f.