United Kingdom
Mathematics Trust

Intermediate Mathematical Challenge

Thursday 7 February 2019

Organised by the United Kingdom Mathematics Trust

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England & Wales: *Year* 11 *or below*
Scotland: *S*4 *or below*
Northern Ireland: *Year* 12 *or below*

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   Each incorrect answer to Questions 16-20 loses 1 mark;
   Each incorrect answer to Questions 21-25 loses 2 marks.
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options.** The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Intermediate Mathematical Challenge should be sent to:

*UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS*2 9*JT*

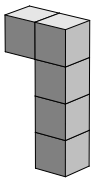
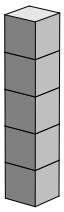
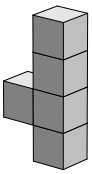
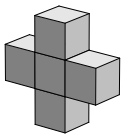
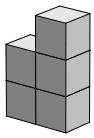
☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

1. What is the value of 2019 tenths?

A 2019 B 201.9 C 20.19 D 2.019 E 0.2019

2. Each of the five shapes shown below is made from five unit cubes.

Which has the smallest surface area?

A  B  C  D  E 

3. There are 120 000 red squirrels living in Scotland. This represents 75% of their total UK population.

How many more red squirrels live in Scotland than live in the remainder of the UK?

A 30 000 B 40 000 C 60 000 D 80 000 E 90 000

4. A 24-hour digital clock shows the time in hours and minutes.

How many times in one day will it display all four digits 2, 0, 1 and 9 in some order?

A 6 B 10 C 12 D 18 E 24

5. The answers to the three calculations below are to be written in descending order.

$X \quad 0.6 \times 0.5 + 0.4$

$Y \quad 0.6 \times 0.5 \div 0.4$

$Z \quad 0.6 \times 0.5 \times 0.4$

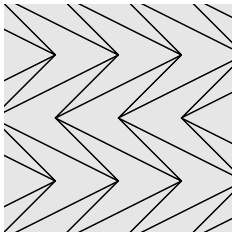
What is the correct order?

A YZX B XYZ C XZY D YXZ E ZYX

6. The diagram shows part of a tessellation of the plane by a quadrilateral.

Khelen wants to colour each quadrilateral in the pattern so that no two quadrilaterals that meet (even at a point) have the same colour.

What is the smallest number of colours he needs?



A 3 B 4 C 5 D 6 E 7

7. How many positive cubes less than 5000 end in the digit 5?

A 1 B 2 C 3 D 4 E 5

8. Three consecutive positive integers less than 20 are, in ascending order, prime, even and triangular.

What is the product of these three integers?

A 6 B 60 C 990 D 1786 E 2730

9. What is the value of $\big(7 - 6 \times (-5)\big) - 4 \times (-3) \div (-2)$?

A -17 B -11 C 31 D 37 E 43

10. A recent report about the amount of plastic created in the last 65 years stated that the 8.3 billion tonnes produced is as heavy as 25 000 Empire State Buildings in New York or a billion elephants.

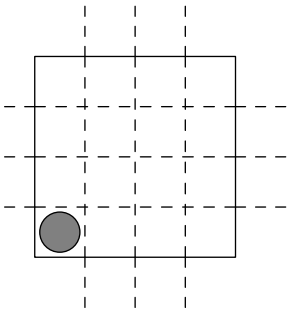
On that basis, how many elephants have the same total weight as the Empire State Building?

A 4000 B 40 000 C 400 000 D 4 000 000 E 40 000 000

11. Which of the following is equal to $\frac{3^9}{9^3}$?

A 3 B 9 C 27 D 81 E 243

12. The game of *Rorrim2* is played on a 4×4 board, starting with a counter in one corner, as shown.



At each turn, the player moves the counter to a cell that is the reflection of its current cell in one of the six dashed lines.

How many cells could the counter occupy after precisely three turns?

A 4 B 6 C 8 D 12 E 16

13. Megan writes down a list of five numbers. The mean of her first three numbers is -3. The mean of her first four numbers is 4. The mean of her first five numbers is -5.

What is the difference between her fourth number and her fifth number?

A 66 B 55 C 44 D 33 E 22

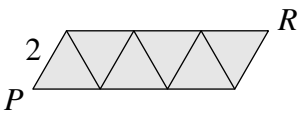
14. There are four people, some of whom always tell the truth. The others always lie.

The first person said, "An odd number of us always tell the truth".
The second person said, "An even number of us always tell the truth".
The third person said, "A prime number of us always tell the truth".
The fourth person said, "A square number of us always tell the truth".

How many of these four people were telling the truth?

A 0 B 1 C 2 D 3 E 4

15. The diagram shows six congruent equilateral triangles, of side-length 2, placed together to form a parallelogram.



What is the length of PR?

A $2\sqrt{13}$ B 7 C $6\sqrt{3}$ D 9 E $7\sqrt{3}$

16. Two numbers x and y have a product which is equal to their sum.

Which of these expressions gives x in terms of y?

A $\frac{y}{y-1}$ B $\frac{y}{y+1}$ C $\frac{y+1}{y}$ D $\frac{y-1}{y}$ E $\frac{y^2}{y+1}$

17. Which of these is equal to $0.\dot{8} + 0.0\dot{7}$?

A $0.8\dot{7}$ B 0.88 C $0.9\dot{5}$ D $0.9\dot{6}$ E $0.9\dot{8}$

18. Two numbers x and y are such that $x + y = \frac{2}{3}$ and $\frac{x}{y} = \frac{2}{3}$.

What is the value of $x - y$?

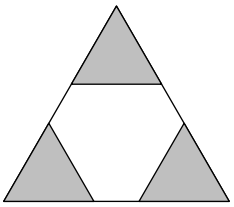
A $-\frac{2}{3}$ B $-\frac{2}{15}$ C $\frac{2}{25}$ D $\frac{2}{5}$ E $\frac{2}{3}$

19. Which of these expressions has the largest value?

A $\frac{1}{2}$ B $\frac{1}{3}+\frac{1}{4}$ C $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ D $\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}$

E $\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}$

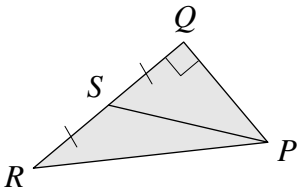
20. Three equilateral triangles with sides of length 1 are shown shaded in a larger equilateral triangle. The total shaded area is half the area of the larger triangle.



What is the side-length of the larger equilateral triangle?

A $\sqrt{5}$ B $\sqrt{6}$ C $\frac{5}{2}$ D $\frac{3\sqrt{3}}{2}$ E $1+\sqrt{3}$

21. The diagram shows a right-angled triangle PQR. The point S is the midpoint of the side QR and $\tan \angle QPR = \frac{3}{2}$.



What is the value of $\sin \angle QPS$?

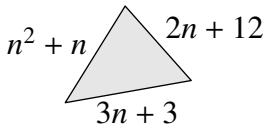
A $\frac{1}{\sqrt{3}}$ B $\frac{1}{\sqrt{2}}$ C $\frac{1}{2}$ D $\frac{3}{5}$ E $\frac{4}{5}$

22. Four of the following six-digit integers are always divisible by 7, regardless of the values of the digits P and Q.

Which of the following is not necessarily a multiple of 7?

A '$PQQ\,PQQ$' B '$PQP\,QPQ$' C '$QPQ\,PQP$' D '$PPP\,PPP$' E '$PPP\,QQQ$'

23. The diagram shows a triangle with sides $n^2 + n$, $2n + 12$ and $3n + 3$.



What is the sum of all the values of n for which the triangle is isosceles?

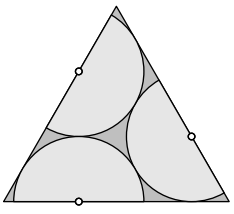
A 7 B 9 C 12 D 13 E 16

24. When 5655 is divided by a two-digit positive integer N, the remainder is 11. When 5879 is divided by the same positive integer N, the remainder is 14.

What is the sum of the digits of N?

A 6 B 7 C 8 D 9 E 10

25. The diagram shows three touching semicircles with radius 1 inside an equilateral triangle, which each semicircle also touches. The diameter of each semicircle lies along a side of the triangle.



What is the length of each side of the equilateral triangle?

A 3 B $\frac{13}{4}$ C $\frac{3}{2}\sqrt{3}$ D $2\sqrt{3}$ E 4

