United Kingdom
Mathematics Trust

Intermediate Mathematical Olympiad

Hamilton paper

Thursday 21 March 2019

Organised by the United Kingdom Mathematics Trust

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England & Wales:	*Year 10*
Scotland:	*S3*
Northern Ireland:	*Year 11*

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

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- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
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- ◇ *Do* not *hand in rough work.*

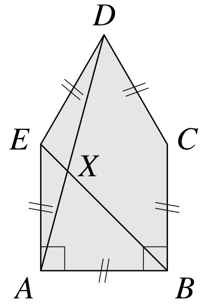
1. A number of couples met and each person shook hands with everyone else present, but not with themselves or their partners.

There were 31 000 handshakes altogether.

How many couples were there?

2. The diagram shows a pentagon $ABCDE$ in which all sides are equal in length and two adjacent interior angles are 90°. The point X is the point of intersection of AD and BE.

Prove that $DX = BX$.



3. A $4\,\text{cm} \times 4\,\text{cm}$ square is split into four rectangular regions using two line segments parallel to the sides.

How many ways are there to do this so that each region has an area equal to an integer number of square centimetres?

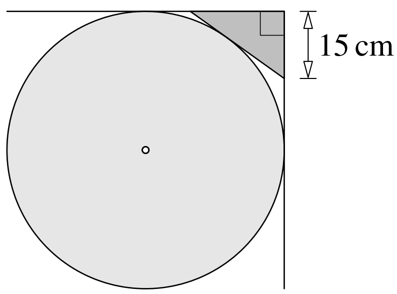
4. Each of A and B is a four-digit palindromic integer, C is a three-digit palindromic integer, and $A - B = C$.

What are the possible values of C?

[*A palindromic integer reads the same 'forwards' and 'backwards'.*]

5. The area of the right-angled triangle in the diagram alongside is $60\,\text{cm}^2$. The triangle touches the circle, and one side of the triangle has length 15 cm, as shown.

What is the radius of the circle?



6. Nine dots are arranged in the 2×2 square grid shown. The arrow points north.

Harry and Victoria take it in turns to draw a unit line segment to join two dots in the grid.

Harry is only allowed to draw an east-west line segment, and Victoria is only allowed to draw a north-south line segment. Harry goes first.

A point is scored when a player draws a line segment that completes a 1×1 square on the grid.

Can either player force a win, no matter how the other person plays?