United Kingdom
Mathematics Trust

Intermediate Mathematical Olympiad

Cayley paper

Thursday 19 March 2020

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England & Wales:	*Year 9 or below*
Scotland:	*S2 or below*
Northern Ireland:	*Year 10 or below*

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

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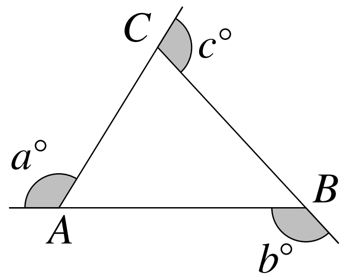
☎ 0113 343 2339   enquiry@ukmt.org.uk   www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but* not *decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do* not *hand in rough work.*

1. In the triangle ABC, the three exterior angles $a°$, $b°$ and $c°$ satisfy $a + b = 3c$.

Prove that the triangle ABC is right-angled.

[Note: The diagram has been included to illustrate the labelling only and is not drawn to scale.]



2. The digits 1, 2, 3, 4, 5, A and B are all different and nonzero. Each of the two six-digit integers 'A12345' and '12345A' is divisible by B.

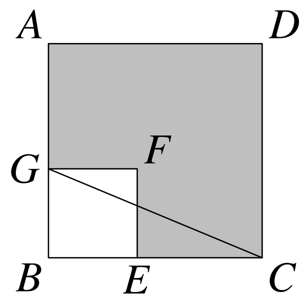
Find all possible pairs of values of A and B.

3. Four friends rent a cottage for a total of £300 for the weekend. The first friend pays half of the sum of the amounts paid by the other three friends. The second friend pays one third of the sum of the amounts paid by the other three friends. The third friend pays one quarter of the sum of the amounts paid by the other three friends.

How much money does the fourth friend pay?

4. Two squares $ABCD$ and $BEFG$ share the vertex B, with E on the side BC and G on the side AB, as shown.
The length of CG is 9 cm and the area of the shaded region is 47 cm^2.

Calculate the perimeter of the shaded region.



5. A ladybird is free to fly between the 1×1 cells of a 10×10 square grid. She may begin in any 1×1 cell of the grid. Every second she flies to a different 1×1 cell that she has not visited before.

Find the smallest number of cells the ladybird must visit, including her starting cell, so that you can be certain that there is a 2×2 grid of adjacent cells, each of which she has visited.

Please turn over for question 6

6. Martha and Nadia play a game. Each has to make her own four-digit number, choosing her four digits from eight "digit cards" labelled 1-8. First Martha chooses her thousands digit, and then Nadia chooses her thousands digit. Next, Martha chooses her hundreds digit from the remaining six cards, and then Nadia chooses her hundreds digit. This process is repeated for the tens and finally the units digits of their numbers. The two four-digit numbers produced are then added together. Martha wins if the sum is not a multiple of 6; Nadia wins if the sum is a multiple of 6.

Determine which player has a winning strategy (that is to say, which player can guarantee that she will win no matter which digits the other player chooses).

