Mathematical Olympiad for Girls

Monday 31st August 2020

Organised by the United Kingdom Mathematics Trust

Instructions

1. This is a sample paper, illustrating the special format for MOG 2020.

2. Time allowed: 2 hours

3. Each question carries 10 marks.

4. Questions 1 to 4 are taken from the 2018 paper and Question 5 from the 2016 paper. Some of the questions have been modified to ask for numerical or algebraic answers rather than proofs. See the 2018 and 2016 papers for the original version of the questions, as well as the solutions.

5. In 2020, you will need to fill out an answer sheet to give to your teacher, who will then copy your answers into an online form.

6. Earlier questions tend to be easier. Some questions have multiple parts. Often earlier parts introduce results or ideas useful in solving later parts of the problem.

1. (a)   (i)   Write down the full factorisation of the expression $a^2 - b^2$.

     (ii)   Find two factors of 359999 (other than 1 and 359999).

    (iii)   Find two factors of 249919 (other than 1 and 249919).

     HINT   You can use your factorisation of $a^2 - b^2$ if you like.   (5 marks)

 (b)   (i)   Write down the full factorisation of $a^2 + 2ab + b^2$.

     (ii)   Find the square root of 9006001.

    (iii)   Find the square root of 11449.

                                                                (5 marks)

2. Triangle ABC is isosceles, with $AB = BC = 1$ and angle ABC equal to $120°$. A circle is tangent to the line AB at A and to the line BC at C.

What is the radius of the circle?   (10 marks)

3. (a) Sheila the snail leaves a trail behind her as she moves along gridlines in Grid 1. She may only move in one direction along a gridline, indicated by arrows. Let b, c, d be the number of different trails Sheila could make while moving from A to B, C, D respectively.

    (i)   Write down an expression for b in terms of c and d.

    (ii)   Find the value of b.

                                               (3 marks)
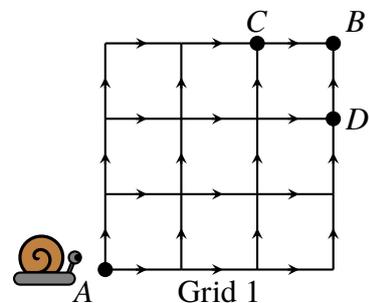
 (b) Ghastly the ghost lives in a haunted mansion with 27 rooms arranged in a $3 \times 3 \times 3$ cube. He may pass unhindered between adjacent rooms, moving through the walls or ceilings. He wants to move from the room in the bottom left corner of the building to the room farthest away in the top right corner, passing through as few rooms as possible. Unfortunately, a trap has been placed in the room at the centre of the house and he must avoid it at all costs.

How many distinct paths through the house can he take?   (7 marks)

4. Each of 100 houses in a row are to be painted white or yellow. The residents are quite particular and request that no three neighbouring houses are all the same colour.

  (a) What is the largest number of houses that can be painted yellow?   (4 marks)

  (b) In how many different ways may the houses be painted if exactly 67 are painted yellow?

                                                                      (6 marks)

5. Alia, Bella and Catherine are multiplying fractions, aiming to obtain integers. Each of them can multiply as many fractions as she likes (including just one), and can use the same fraction more than once.

Alia's fractions are of the form $\dfrac{n+1}{n}$, where n is a positive integer.

Bella's fractions are of the form $\dfrac{6p-5}{3p+6}$, where p is a positive integer.

Catherine's fractions are of the form $\dfrac{4q-1}{2q+1}$, where q is a positive integer.

How many integers between 1 and 100 inclusive

  (a) can Alia obtain?   (2 marks)

  (b) can Bella obtain?   (3 marks)

  (c) can Catherine obtain?   (5 marks)

Answer Sheet for MOG 2020 Sample Paper

This is the answer sheet to go with the MOG Sample Paper for 2020. In MOG 2020, you will fill out an answer sheet (on paper) and hand it to your teacher, who will then transfer them into an online form. It is important that you follow the instructions on the answer sheet carefully.

Question	Part		Your answer
1.	**a**	(i)	
		(ii)	
		(iii)	
	b	(i)	
		(ii)	
		(iii)	
2.			
3.	**a**	(i)	
		(ii)	
	b		
4.	**a**		
	b		
5.	**a**		
	b		
	c		