United Kingdom
Mathematics Trust

Junior Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	C	D	C	B	D	E	E	D	D	D	E	E	C	D	C	A	B	C	E	A	A	D	D	E

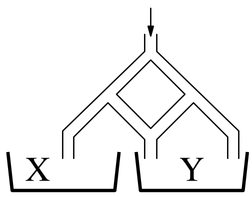
1. Which of these expressions has the largest value?

A $1 + 2 \times 3 + 4$ B $1 + 2 + 3 \times 4$ C $1 + 2 + 3 + 4$ D $1 \times 2 + 3 + 4$ E $1 \times 2 + 3 \times 4$

Solution **B**

In this question we must remember to do the multiplications before any of the additions. Hence the values of the five expressions are, in order, 11, 15, 10, 9 and 14. Therefore the expression with the largest value is B.

2. Lily pours 296 litres of water into the top of the pipework shown in the diagram. Each time a pipe forks, half the water flows to one side and half to the other. How many litres of water will reach container Y?



A 210 B 213 C 222 D 225 E 231

Solution **C**

Consider first the amount of water that will reach X. Since half the water flows to each side each time a pipe forks, the amount of water, in litres, reaching X is $\frac{1}{2} \times \frac{1}{2} \times 296 = 74$. Therefore the amount of water, in litres, reaching Y is $296 - 74 = 222$.

3. Andrew wants to write the letters of the word KANGAROO in the cells of a 2×4 grid such that each cell contains exactly one letter. He can write the first letter in any cell he chooses but each subsequent letter can only be written in a cell with at least one common vertex with the cell in which the previous letter was written. Which of the following arrangements of letters could he not produce in this way?

A

K	A
N	O
O	G
R	A

B

N	G
A	A
K	R
O	O

C

O	O
K	R
A	A
G	N

D

K	A
N	G
O	O
R	A

E

K	O
A	O
R	N
A	G

Solution **D**

To produce the arrangement of diagram D, Andrew would first need to write the letters K, A, N and G in the top four cells as shown. He would then need to write A in a vacant cell next to the G. Therefore he could not write O and O in the third row. Hence arrangement D could not be produced in the way described.

(It is left as an exercise for readers to show that all the other arrangements can be produced in the way described.)

4. At 8:00 my watch was four minutes slow. However, it gains time at a constant rate and at 16:00 on the same day it was six minutes fast. At what time did it show the correct time?

A 9:10 B 10:11 C 11:12 D 12:13 E 13:14

Solution **C**

In the 8 hours, or 480 minutes, from 8:00 to 16:00, my watch gains 10 minutes. Since it is 4 minutes slow at 8:00, it will show the correct time in $\frac{4}{10}$ of 480 minutes after 8:00. Hence it will show the correct time 192 minutes after 8:00, which is 11:12.

5. In how many two-digit numbers is one digit twice the other?

A 6 B 8 C 10 D 12 E 14

Solution **B**

The two-digit integers where one digit is twice the other are 12 and 21, 24 and 42, 36 and 63, 48 and 84. Therefore there are eight such numbers.

6. What day will it be 2021 hours after 20:21 on Monday?

A Friday B Thursday C Wednesday D Tuesday E Monday

Solution **D**

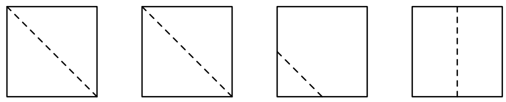
First note that $2021 = 84 \times 24 + 5$. Therefore 2021 hours after 20:21 on a Monday is 84 days and 5 hours later. Since 84 days is equal to 12 weeks, this will be 5 hours after 20:21 on a Monday and hence is at 01:21 on a Tuesday.

7. A square of paper is cut into two pieces by a single straight cut.
Which of the following shapes *cannot* be the shape of either piece?

A An isosceles triangle
B A right-angled triangle
C A pentagon
D A rectangle
E A square

Solution **E**

Assuming one of the options cannot be formed, we answer the question by eliminating four of the options. The diagrams below show examples of how a square could be cut to form the first four shapes, with a dotted line representing the position of the cut.



Hence we conclude that the shape which cannot be obtained is a square.

8. When Cathy the cat just lazes around, she drinks 60 ml of milk per day. However, each day she chases mice she drinks a third more milk. Also, each day she gets chased by a dog she drinks half as much again as when she chases mice. In the last two weeks Cathy has been chasing mice on alternate days and has also been chased by a dog on two other days. How much milk did she drink in the last two weeks?

A 900 ml B 980 ml C 1040 ml D 1080 ml E 1100 ml

Solution **E**

On a day Cathy chases mice, the amount of milk she drinks, in ml, is $(60 + \frac{1}{3} \times 60) = 80$. On a day when she gets chased by a dog, the amount of milk she drinks, in ml, is $(80 + \frac{1}{2} \times 80) = 120$. Therefore, in the last two weeks, the amount of milk Cathy has drunk, in ml, is $(7 \times 80 + 2 \times 120 + 5 \times 60) = 1100$.

9. The houses on the south side of Crazy Street are numbered in increasing order starting at 1 and using consecutive odd numbers, except that odd numbers that contain the digit 3 are missed out. What is the number of the 20th house on the south side of Crazy Street?

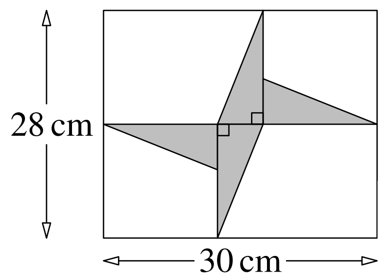
A 41 B 49 C 51 D 59 E 61

Solution **D**

The house numbers on the south side of Crazy Street are consecutive odd numbers apart from any which contain the digit 3. Therefore the numbers start 1, 5, 7, 9, 11, 15, 17, 19, etc. It can be seen that the tens digit increases every four houses. Hence the 20th house would be the last house of the fifth group of four houses. Since no house numbers contain the digit 3, there is no group of houses whose house numbers start with a 3 and hence the numbers in the fifth group of four houses start with a 5. Therefore the number of the 20th house is 59.

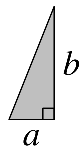
10. The diagram shows four congruent right-angled triangles inside a rectangle. What is the total area, in cm^2, of the four triangles?

A 46 B 52 C 54 D 56 E 64



SOLUTION **D**

Let the lengths of the two shorter sides of the right-angled triangles be a cm and b cm as shown in the diagram. From the diagram in the question, it can be seen that $2b = 28$ and $a + 2b = 30$, and hence $b = 14$ and $a = 2$. Therefore the total area, in cm^2, of the four triangles is $4 \times (\frac{1}{2} \times 2 \times 14) = 56$.



11. Dad says he is exactly 35 years old, not counting weekends. How old is he really?

A 40 B 42 C 45 D 49 E 56

SOLUTION **D**

Since Dad is excluding weekends, he is only counting $\frac{5}{7}$ of the days. Therefore 35 is $\frac{5}{7}$ of his real age. Hence his real age is $35 \times \frac{7}{5} = 49$.

12. The mean of a set of 8 numbers is 12. Two numbers with a mean of 18 are removed from the set. What is the mean of the remaining 6 numbers?

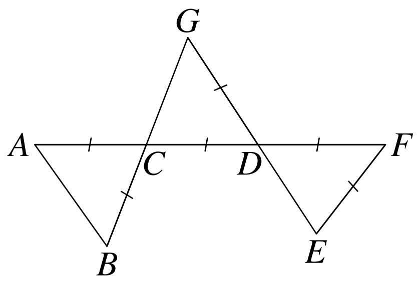
A 6 B 7 C 8 D 9 E 10

SOLUTION **E**

The total of the set of 8 numbers is $8 \times 12 = 96$. The total of the two numbers removed is $2 \times 18 = 36$. Therefore the total of the 6 remaining numbers is $96 - 36 = 60$. Hence the mean of the 6 remaining numbers is $60 \div 6 = 10$.

13. The diagram shows three triangles which are formed by the five line segments $ACDF$, BCG, GDE, AB and EF so that $AC = BC = CD = GD = DF = EF$. Also $\angle CAB = \angle EFD$.
What is the size, in degrees, of $\angle CAB$?

A 40 B 45 C 50 D 55 E 60



Solution **E**

Let the size in degrees of $\angle CAB$ be x. Since $AC = BC$, triangle ABC is isosceles. Hence $\angle ABC = x°$. Since angles in a triangle add to 180°, we have $\angle BCA = (180 - 2x)°$. Also, since vertically opposite angles are equal, we have $\angle GCD = (180 - 2x)°$.

The same argument can then be applied to triangle CGD which is isosceles since $CD = GD$. Hence $\angle CDG = (180 - 2(180 - 2x))° = (4x - 180)°$. Therefore, since vertically opposite angles are equal, $\angle FDE = (4x - 180)°$.

The same argument can be applied once more, this time to triangle FDE, which is also isosceles since $DF = EF$. This gives $\angle EFD = (180 - 2(4x - 180))° = (540 - 8x)°$.

However, we are also told that $\angle CAB = \angle EFD$ and hence $x = 540 - 8x$ or $9x = 540$. This has solution $x = 60$ and hence the size, in degrees, of $\angle CAB$ is 60.

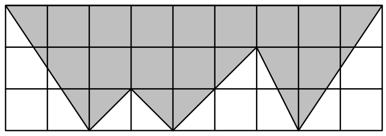
14. The ratio $a : b : c = 2 : 3 : 4$. The ratio $c : d : e = 3 : 4 : 5$. What is the ratio $a : e$?

A 1 : 10 B 1 : 5 C 3 : 10 D 2 : 5 E 1 : 2

Solution **C**

The ratio $a : b : c = 2 : 3 : 4 = 6 : 9 : 12$.
The ratio $c : d : e = 3 : 4 : 5 = 12 : 16 : 20$.
Therefore the ratio $a : b : c : d : e = 6 : 9 : 12 : 16 : 20$.
Hence the ratio $a : e = 6 : 20 = 3 : 10$.

15. Each square in the grid shown is 1 cm by 1 cm. What is the area of the shaded figure, in cm^2?



A 14 B 15 C 16 D 17 E 18

Solution **D**

The area of the grid, in cm^2, is $3 \times 9 = 27$. The unshaded region consists of four triangles. Therefore the area of the unshaded region, in cm^2, is
$\frac{1}{2} \times 2 \times 3 + \frac{1}{2} \times 2 \times 1 + \frac{1}{2} \times 3 \times 2 + \frac{1}{2} \times 2 \times 3 = 3 + 1 + 3 + 3 = 10$.
Therefore the area of the shaded figure, in cm^2, is $27 - 10 = 17$.

16. Aimee says Bella is lying. Bella says Marc is lying. Marc says Bella is lying. Terry says Aimee is lying. How many of the four children are lying?

A 0 B 1 C 2 D 3 E 4

Solution **C**

First, assume Aimee is telling the truth. Then Bella is lying, Marc is telling the truth and Terry is lying.
Second, assume Aimee is lying. Then Bella is telling the truth, Marc is lying and Terry is telling the truth.
Although it is impossible to tell precisely who is lying from the information given, in each case two children are lying.

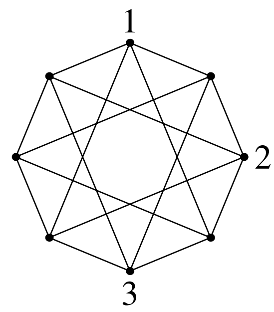
17. In three games a football team scored three goals and conceded one. In those three games, the club won one game, drew one game and lost one game. What was the score in the game they won?

A 3 - 0 B 2 - 0 C 1 - 0 D 3 - 1 E 2 - 1

Solution **A**

The team only conceded one goal. Therefore, since they lost one game, the score in that game was 0 - 1. The team also drew one game and, since they did not concede a goal in the drawn game, the score was 0 - 0. Therefore, since they scored three goals, the score in the game they won was 3 - 0.

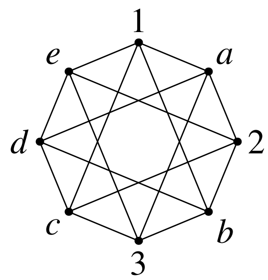
18. The diagram shows the eight vertices of an octagon connected by line segments. Jodhvir wants to write one of the integers 1, 2, 3 or 4 at each of the vertices so that the two integers at the ends of every line segment are different. He has already written three integers as shown.
How many times will the integer 4 appear in his completed diagram?



A 5 B 4 C 3 D 2 E 1

Solution **B**

Let the integers at the vertices of Jodhvir's completed diagram be as shown.



The vertices where a, b, c and e are written are all joined by line segments to vertices labelled 1, 2 and 3. Therefore, since the two integers at the ends of any line segment are different, each of a, b, c and e is equal to 4. The vertex where d is written is joined to the vertices where a, b, c and e are written and hence is different to a, b, c and e. Therefore the integer 4 will appear four times in his completed diagram.

19. Sacha places 25 counters into 14 boxes so that each box contains 1, 2 or 3 counters. No box is inside any other box. Seven boxes contain 1 counter. How many contain 3 counters?

A 2 B 3 C 4 D 5 E 6

Solution **C**

Since seven boxes contain one counter, the remaining seven boxes contain 18 counters. Let the number of boxes containing 3 counters be x. Therefore $3x + 2(7 - x) = 18$. Hence $3x + 14 - 2x = 18$ which has solution $x = 4$. Therefore four boxes contain 3 counters.

20. In the addition sum shown, J, K and L stand for different digits.
What is the value of $J + K + L$?

$$\begin{array}{r} J\,K\,L \\ J\,L\,L \\ +\ J\,K\,L \\ \hline 4\ 7\ 9 \end{array}$$

A 6 B 8 C 9 D 10 E 11

Solution **E**

When we consider the units column, we have $3L = 9$ or $3L = 19$. Since only 9 is divisible by 3, $L = 3$. Then when we consider the tens column, we have $2K + 3 = 7$ or $2K + 3 = 17$. Therefore $K = 2$ or $K = 7$. However, if $K = 2$, the hundreds column would tell us that $3J = 4$, which is not possible. Hence $K = 7$. Now, when we consider the hundreds column, we have $3J + 1 = 4$, which has solution $J = 1$. Therefore the value of $J + K + L$ is $1 + 7 + 3 = 11$.

21. In a particular month there were 5 Saturdays and 5 Sundays but only 4 Mondays and 4 Fridays.
What must occur in the next month?

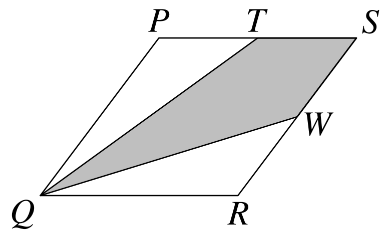
A 5 Wednesdays B 5 Thursdays C 5 Fridays D 5 Saturdays E 5 Sundays

Solution **A**

A month with 5 Saturdays and 5 Sundays but only 4 Mondays and 4 Fridays contains 30 days and ends on a Sunday. Each month in the year that contains 30 days is followed immediately by a month containing 31 days. Therefore the next month contains 31 days and starts on a Monday. Hence it will contain 5 Mondays, 5 Tuesdays and 5 Wednesdays but only 4 of the other four days.

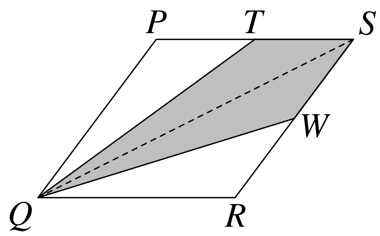
22. In the diagram $PQRS$ is a rhombus. Point T is the mid-point of PS and point W is the mid-point of SR. What is the ratio of the unshaded area to the shaded area?

A 1 : 1 B 2 : 3 C 3 : 5 D 4 : 7 E 5 : 9



Solution **A**

First draw in the line segment QS as shown. Since we are told that T is the mid-point of PS, the triangles PTQ and TSQ have equal bases. Since they also have the same perpendicular height, their areas are equal.

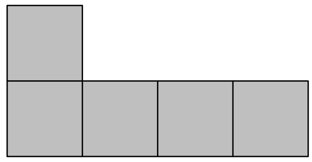


Similarly, since W is the mid-point of SR, we have the areas of triangles SWQ and WRQ being equal. Hence the unshaded area, which is equal to the sum of the areas of triangles PTQ and WRQ, is the same as the shaded area, which is equal to the sum of the areas of triangles TSQ and SWQ.

Therefore the ratio of the unshaded area to the shaded area is 1 : 1.

23. Using only pieces like the one shown in the diagram, Zara wants to make a complete square without gaps or overlaps. What is the smallest number of pieces she can use?

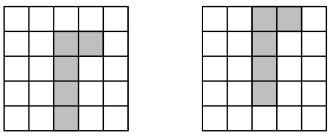
A 5 B 8 C 16 D 20 E 75



SOLUTION **D**

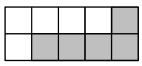
Since the total area of a square created with N of the pieces shown will be $5N$, we need $5N$ to be a square number. Therefore N is of the form $5m^2$ for some integer m and, of the options given, only 5 and 20 are of that form.

Consider first whether it is possible to create a square with 5 pieces. Such a square would be a 5×5 square and, as shown in the diagrams below, it is then only possible to cover the central cell in one of two ways (rotations and reflections of the arrangements shown being essentially the same).



Whichever of these two arrangements is used to cover the central cell, it is easy to see that the remaining cells cannot be covered with four more of the pieces and hence it is impossible to build a square with 5 of the pieces shown.

However, since two of the pieces can be placed next to each other to form a 5×2 rectangle as shown,



and 10 of these rectangles can easily be combined to create a 10×10 square, a square can be built using 20 pieces.

24. Four positive numbers p, q, r and s are in increasing order of size. One of the numbers is to be increased by 1 so that the product of the four new numbers is now as small as possible.
Which number should be increased?

A p B q C r D s E either q or r

SOLUTION **D**

The product of the four numbers is $pqrs$. If each number is increased by 1 in turn, the new products are $(p+1)qrs = pqrs + qrs$, $p(q+1)rs = pqrs + prs$, $pq(r+1)s = pqrs + pqs$ and $pqr(s+1) = pqrs + pqr$. Hence, to minimise the new product, we need to include the minimum of qrs, prs, pqs and pqr. Since p, q, r and s are in increasing order, the minimum value of these is pqr. Therefore the number which should be increased by 1 is s.

25. Sonia wants to write a positive five-digit integer whose digits are 1, 2, 3, 4 and 5 in some order. The first digit of the integer is to be divisible by 1, the first two digits are to form a two-digit integer divisible by 2, the first three digits are to form a three-digit integer divisible by 3, the first four digits are to form a four-digit integer divisible by 4 and the five-digit integer itself is to be divisible by 5. How many such five-digit integers could Sonia write?

A 10 B 5 C 2 D 1 E 0

Solution **E**

Suppose such a five-digit integer exists. Since it is to be divisible by 5, its last digit must be 5. The two-digit integer consisting of the first two digits and the four-digit integer consisting of the first four digits are to be divisible by 2 and by 4 respectively. Therefore the second and the fourth digits of the five-digit integer are both even. Therefore the five-digit integer is of the form *2*45 or *4*25, with the missing digits being 1 and 3 in some order. The three-digit integer made up of the first three digits is to be divisible by 3 and hence the sum of the first three digits should be divisible by 3. However neither 143 nor 341 is divisible by 3 and hence the five-digit integer is of the form *2*45. But neither 1234 nor 3214 is divisible by 4. Therefore there are no possible five-digit integers that Sonia could write.