United Kingdom Mathematics Trust

Grey Kangaroo

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	A	A	B	B	D	E	C	B	D	C	B	C	E	C	C	B	D	D	A	C	A	B	A	C

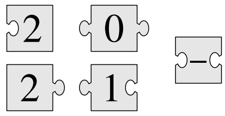
1. What is the value of $\dfrac{20 \quad 21}{2 ¸ 0 ¸ 2 ¸ 1}$?

A 42 B 64 C 80 D 84 E 105

Solution **D**

The expression can be simplified to $\frac{20 \quad 21}{5} = 4 \quad 21 = 84$•

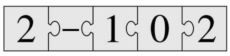
2. When the five pieces shown are fitted together correctly, the result is a rectangle with a calculation written on it. What is the answer to this calculation?



A 100 B 8 C 1 D 199 E 208

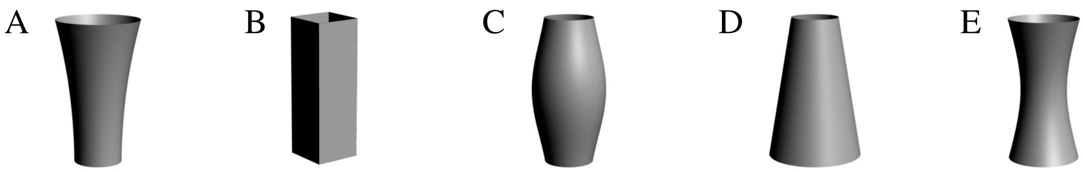
Solution **A**

When you look at the pieces, you can see that the calculation both starts and ends with a piece with a “2” written on. Further, the only piece that can be placed next to the first “2” is the subtraction sign. Then the “0” must be attached to the final “2” and so the rectangle must be as shown.



Hence the answer to the calculation is -100.

3. Each of the five vases shown has the same height and each has a volume of 1 litre. Half a litre of water is poured into each vase. In which vase would the level of the water be the highest?



Solution **A**

Since each of the vases B, C and E has a horizontal line of symmetry, half a litre of water would fill each vase to a level equal to half its height. In vase D the water level would be below half the height whereas in vase A the water level would be above half the height. Hence the answer is A.

4. A student correctly added the two two-digit numbers on the left of the board and got the answer 137. What answer will she obtain if she adds the two four-digit numbers on the right of the board?

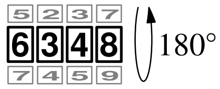
$$\begin{array}{r} AB \\ +CD \\ \hline 137 \end{array} \qquad \begin{array}{r} ADCB \\ +CBAD \\ \hline ? \end{array}$$

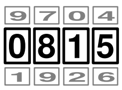
A 13737 B 13837 C 14747 D 23723 E 137137

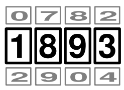
Solution **B**

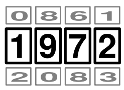
Since the sum of the two two-digit numbers and is 137, the sum of the two-digit numbers and is also 137. Therefore the sum of the two four-digit numbers and is (100 +) + (100 +) = 100(+) + (+) = 100 137 + 137 = 13700 + 137 = 13837.

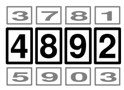
5. A bike lock has four wheels numbered with the digits 0 to 9 in order. Each of the four wheels is rotated by 180 from the code shown in the first diagram to get the correct code. What is the correct code for the bike lock?

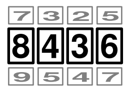


A 

B 

C 

D 

E 

Solution **B**

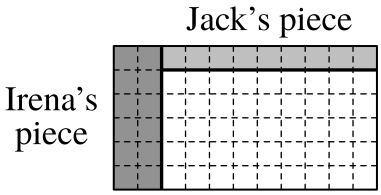
Since each wheel has 10 digits on it, rotating any wheel by 180° will increase or decrease the value of the digit visible by 5. The digits currently showing are 6, 3, 4 and 8 and hence the correct code is 1893.

6. A rectangular chocolate bar is made of equal squares. Irena breaks off two complete strips of squares and eats the 12 squares she obtains. Later, Jack breaks off one complete strip of squares from the same bar and eats the 9 squares he obtains. How many squares of chocolate are left in the bar?

A 72 B 63 C 54 D 45 E 36

Solution **D**

Since Irena breaks off two strips containing 12 squares of chocolate, one strip contains six squares. Since Jack breaks off one strip containing nine squares, he must have broken his strip from a longer side of the bar while Irena broke her strips from a shorter side of the bar. Therefore the remaining bar is nine squares long and five squares wide and hence now contains 45 squares of chocolate, as illustrated in the diagram below.



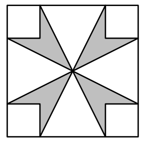
7. When a jar is one-fifth filled with water, it weighs 560 g. When the same jar is four-fifths filled with water, it weighs 740 g. What is the weight of the empty jar?

A 60 g B 112 g C 180 g D 300 g E 500 g

Solution **E**

The information in the question tells us that the amount of water that would fill three-fifths of the jar weighs $(740 - 560)$ g = 180 g. Therefore the weight of water that would fill one-fifth of the jar is $(180 \div 3)$ g = 60 g. Hence the weight of the empty jar is $(560 - 60)$ g = 500 g.

8. In the diagram, the area of the large square is 16 cm^2 and the area of each small corner square is 1 cm^2. What is the shaded area?



A 3 cm^2 B $\frac{7}{2}$ cm^2 C 4 cm^2 D $\frac{11}{2}$ cm^2 E 6 cm^2

Solution **C**

Since the area of the large square is 16 cm^2 and the area of each small square is 1 cm^2, their side-lengths are 4 cm and 1 cm respectively. Therefore the base of each of the four triangles is 2 cm and, since these triangles meet at the centre of the large square, the height of each triangle is also 2 cm. Therefore the total area of the four triangles is $(4 \times \frac{1}{2} \times 2 \times 2)\,\text{cm}^2 = 8\,\text{cm}^2$. Hence the shaded area is $(16 - 8 - 4 \times 1)\,\text{cm}^2 = 4\,\text{cm}^2$.

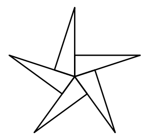
9. Costa is building a new fence in his garden.
He uses 25 planks of wood, each of which is 30 cm long.
He arranges these planks so that there is the same slight overlap between any two adjacent planks, as shown in the diagram. The total length of Costa's new fence is 6.9 metres. What is the length in centimetres of the overlap between any pair of adjacent planks?

A 2.4 B 2.5 C 3 D 4.8 E 5

Solution **B**

Let the length of the overlap be H cm. From the diagram in the question, it can be seen that the total length of the fence can be calculated as the total length of the 13 pieces in the lower row in the diagram and the total length of the 12 planks in the upper row of the diagram, with each of the 12 having two overlaps removed. Hence $690 = 13 \times 30 + 12 \times (30 - 2H)$. Therefore $690 = 390 + 360 - 24H$ and hence $24H = 60$. This has solution $H = 2.5$. Therefore the overlap between adjacent planks is 2.5 cm.

10. Five identical right-angled triangles can be arranged so that their larger acute angles touch to form the star shown in the diagram. It is also possible to form a different star by arranging more of these triangles so that their smaller acute angles touch.
How many triangles are needed to form the second star?

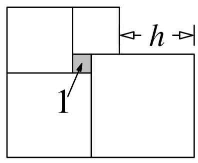


A 10 B 12 C 18 D 20 E 24

SOLUTION **D**

Since the five identical triangles meet at a point, the size of the larger acute angle in each triangle is 360° 5 = 72°. Therefore the smaller acute angle in each triangle is 180° 90° 72° = 18°. Hence, since the second star is formed using the triangles whose smaller acute angles touch, the number of triangles needed to form the second star is 360 18 = 20.

11. Five squares are positioned as shown. The small square indicated has area 1.
What is the value of ?

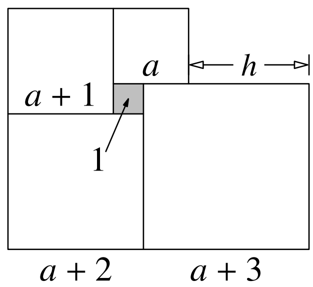


A 3 B 3.5 C 4 D 4.2 E 4.5

SOLUTION **C**

Since the shaded square has area 1, its side-length is 1. Let the side-length of the square above the shaded square be 0, as shown in the diagram.

Therefore the side-lengths of the other squares, going anti-clockwise, are 0 ¸ 1, 0 ¸ 1 ¸ 1 = 0 ¸ 2 and 0 ¸ 2 ¸ 1 = 0 ¸ 3. From the diagram, it can be seen that 1 ¸ 0 ¸ 3 = 0 ¸ and hence the value of is 4.



12. There are 20 questions in a quiz. Seven points are awarded for each correct answer, four points are deducted for each incorrect answer and no points are awarded or deducted for each question left blank.

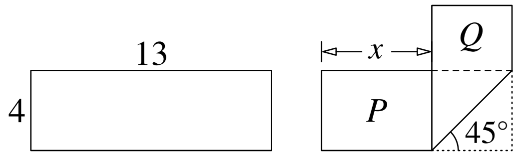
Erica took the quiz and scored 100 points. How many questions did she leave blank?

A 0 B 1 C 2 D 3 E 4

SOLUTION **B**

Let the number of correct answers Erica gave be and the number of wrong answers be , . Since the total number of marks Erica obtained for the quiz is 100, we have 7 4 , = 100 or 7 = 100 ¸ 4 , . Therefore is a multiple of 4 greater than 14 and smaller than 20, since clearly not all answers were correct. Hence = 16, which corresponds to , = 3. Therefore the number of blanks is 20 16 3 = 1.

13. A rectangular strip of paper of dimensions 4 13 is folded as shown in the diagram. Two rectangles are formed with areas % and & where % = 2&. What is the value of G?



A 5 p_ B 5.5 C 6 D 6.5
E 4 2

Solution **C**

Let the height of the rectangle & be H. Since the original 4 13 rectangle has been folded to form the second shape, both the width of the rectangle with area & and the height of the rectangle with area % are 4. Considering the base of the rectangle before and after folding gives the equation G ¸ 4 ¸ H = 13 and hence G ¸ H = 9. Since the two rectangles both have one side of length 4, the condition % = 2& implies that G = 2H and so H = 3 and G = 6.

14. A box of fruit contained twice as many apples as pears. Chris and Lily divided them up so that Chris had twice as many pieces of fruit as Lily. Which one of the following statements is always true?

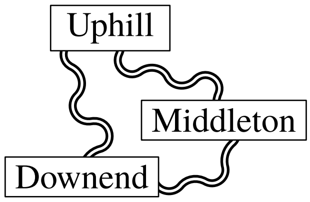
A Chris took at least one pear.
B Chris took twice as many apples as pears
C Chris took twice as many apples as Lily.
D Chris took as many apples as Lily took pears.
E Chris took as many pears as Lily took apples.

Solution **E**

Let the total number of pieces of fruit in the box be 3G. Since Chris took twice as many pieces of fruit as Lily, he took 2G pieces in total and Lily took G pieces in total. Also, since the box contained twice as many apples as pears, there were 2G apples and G pears in total. Let the number of pears Chris took be H. Therefore, since he took 2G pieces of fruit in total, Chris took 2G H apples, leaving H apples for Lily. Hence the number of apples Lily took is always the same as the number of pears Chris took.

Note: although the argument above shows that option is always true, it does not show that the others are not. Consider the case where the box contains 2 apples and 1 pear. Chris's options are to take 2 apples, leaving Lily with 1 pear or to take 1 apple and 1 pear, leaving Lily with 1 apple. In the first instance, options , , and are all untrue and hence none of these can always be true.

15. Three villages are connected by paths as shown. From Downend to Uphill, the detour via Middleton is 1 km longer than the direct path. From Downend to Middleton, the detour via Uphill is 5 km longer than the direct path. From Uphill to Middleton, the detour via Downend is 7 km longer than the direct path. What is the length of the shortest of the three direct paths between the villages?



A 1 km B 2 km C 3 km D 4 km E 5 km

Solution **C**

Let the lengths of the direct paths from Uphill to Middleton, Middleton to Downend and Downend to Uphill be G km, H km and I km respectively. The information in the question tells us that G ¸ H = I ¸ 1, G ¸ I = H ¸ 5 and H ¸ I = G ¸ 7. When we add these three equations, we obtain 2G ¸ 2H ¸ 2I = I ¸ H ¸ G ¸ 13 and hence G ¸ H ¸ I = 13. Therefore 13 = 2I ¸ 1, 13 = 2H ¸ 5 and 13 = 2G ¸ 7, which have solutions I = 6, H = 4 and G = 3. Hence the length of the shortest of the direct paths is the one from Uphill to Middleton with length 3 km.

16. In a particular fraction the numerator and denominator are both positive. The numerator of this fraction is increased by 40%. By what percentage should its denominator be decreased so that the new fraction is double the original fraction?

A 10% B 20% C 30% D 40% E 50%

Solution **C**

Let the original fraction be $\frac{G}{H}$. Since the numerator of the new fraction is obtained by increasing the numerator of the old fraction by 40%, its value is 1•4G. Let the denominator of the new fraction be :H. We are told that the new fraction is twice the old fraction and hence $\frac{1•4G}{:H}$ = 2¹$\frac{G}{H}$º. Therefore $\frac{1•4}{:}$ = 2 and hence : = 0•7. Therefore the denominator of the new fraction is 70% of the denominator of the original fraction and hence has been decreased by 30%.

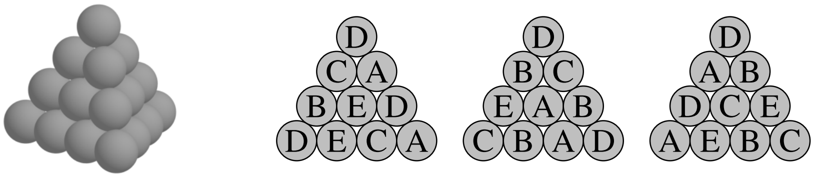
17. The six-digit number 2%&'() is multiplied by 3 and the result is the six-digit number %&'()2. What is the sum of the digits of the original number?

A 24 B 27 C 30 D 33 E 36

Solution **B**

Let the five-digit number %&'() be G. The condition in the question tells us that 3¹200 000 ¸ Gº = 10G ¸ 2. Therefore 600 000 ¸ 3G = 10G ¸ 2 and hence 599 998 = 7G. This has solution G = 85 714 and hence the sum of the digits of the original number is 2 ¸ 8 ¸ 5 ¸ 7 ¸ 1 ¸ 4 = 27.

18. A triangular pyramid is built with 20 cannonballs, as shown.



Each cannonball is labelled with one of A, B, C, D or E. There are four cannonballs with each type of label.The diagrams show the labels on the cannonballs on three of the faces of the pyramid. What is the label on the hidden cannonball in the middle of the fourth face?

A B C D E

Solution **D**

Note that each cannonball on the two non-horizontal edges of each pictured face appears on two of those faces, except the cannonball at the vertex which appears on all three. Hence, when the labels of the cannonballs are counted, these must only be counted once. Careful counting of the cannonballs shown gives four cannonballs labelled , , and but only three labelled . Hence the cannonball at the centre of the hidden face is labelled .

19. A ball is made of white hexagons and black pentagons, as seen in the picture. There are 12 pentagons in total. How many hexagons are there?

A 12 B 15 C 18 D 20 E 24

Solution **D**

From the diagram in the question, it can be seen that each pentagon shares an edge with five hexagons and that each hexagon shares an edge with three different pentagons.
Therefore the total number of hexagons is 12 5 3 = 20.

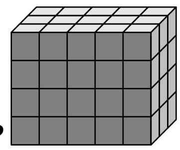
20. The positive integer # is the smallest one whose digits add to 41. What is the sum of the digits of # ¸ 2021?

A 10 B 12 C 16 D 2021 E 4042

Solution **A**

The smallest positive integer # whose digits add to 41 has as few digits as possible and then as small a digit as possible as its first digit. Hence as many digits as possible, except for the first digit, should be 9. Since 41 = 4 9 ¸ 5, we have # = 59 999. Therefore the value of # ¸ 2021 is 62 020 with digit sum 6 ¸ 2 ¸ 2 = 10.

21. The diagram shows a $3 \times 4 \times 5$ cuboid consisting of 60 identical small cubes. A termite eats its way along the diagonal from P to Q. This diagonal does not intersect the edges of any small cube inside the cuboid. How many of the small cubes does it pass through on its journey?



A 8 B 9 C 10 D 11 E 12

Solution **C**

We are told that the diagonal from P to Q does not intersect any internal edges. So, while it goes from P to Q, it moves from one small cube to another by passing through a face. It will have to pass through at least 3 faces in order to get from the base layer up to the top layer, through 4 faces to get from the left to the right and 2 faces to get from the front to the back. The termite starts in the small cube at P and then must pass through another $3 + 4 + 2$ new small cubes to reach Q. In total, therefore, it must pass through 10 small cubes.

22. Lewis and Geraint left Acaster to travel to Beetown at the same time. Lewis stopped for an hour in Beetown and then drove back towards Acaster. He drove at a constant 70 km/h. He met Geraint, who was cycling at a constant 30 km/h, 105 km from Beetown. How far is it from Acaster to Beetown?

A 315 km B 300 km C 250 km D 210 km E 180 km

Solution **A**

Let the distance from Acaster to Beetown be x km. Lewis met Geraint 105 km from Beetown. Therefore, when they met, Lewis had travelled $(x + 105)$ km and Geraint had travelled $(x - 105)$ km. Since time $= \frac{\text{distance}}{\text{speed}}$, when they met, Geraint had been travelling for $\left(\frac{x-105}{30}\right)$ hours and Lewis had been travelling for $\left(\frac{x+105}{70} + 1\right)$ hours, where the extra '+1' represents the hour Lewis spends in Beetown. These two times are equal and hence $\frac{x-105}{30} = \frac{x+105}{70} + 1$. When we multiply each term in this equation by 210, we get $7(x - 105) = 3(x + 105) + 210$. This can be simplified to $4x = 1260$, which has solution $x = 315$.

23. A total of 2021 coloured koalas are arranged in a row and are numbered from 1 to 2021. Each koala is coloured red, white or blue. Amongst any three consecutive koalas, there are always koalas of all three colours. Sheila guesses the colours of five koalas. These are her guesses: Koala 2 is white; Koala 20 is blue; Koala 202 is red; Koala 1002 is blue; Koala 2021 is white. Only one of her guesses is wrong. What is the number of the koala whose colour she guessed incorrectly?

A 2 B 20 C 202 D 1002 E 2021

Solution **B**

Since the question tells us that amongst any three consecutively numbered koalas, there are always koalas of all three colours, the colours of the koalas will repeat every three koalas. The numbers of the koalas we have information about are 2, $20 = 6 \times 3 + 2$, $202 = 67 \times 3 + 1$, $1002 = 334 \times 3$ and $2021 = 673 \times 3 + 2$. It can be seen that 2, 20 and 2021 all give the same remainder when divided by 3 and hence, since the colours repeat every three koalas, these koalas should be the same colour. However, koala 20 is guessed to be blue, whereas koalas 2 and 2021 are both guessed to be white, so it is koala 20 whose colour has been guessed incorrectly.

24. In a tournament each of the six teams plays one match against every other team. In each round of matches, three take place simultaneously. A TV station has already decided which match it will broadcast for each round, as shown in the diagram. In which round will team S play against team U?

1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R

A 1 B 2 C 3 D 4 E 5

SOLUTION **A**

Consider team %. We are told the timing of three of its matches, against teams &,) and ' in rounds 1, 3 and 5 respectively. This leaves fixtures against teams * and (to be fixed in rounds 2 or 4 and, since we are given that team * is due to play team) in round 4, team % plays team (in round 4 and team * in round 2. The missing fixtures in rounds 2 and 4 can then be added to give the partial fixture list shown below.

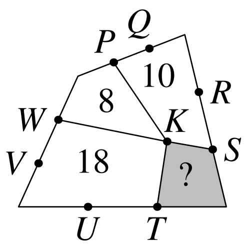
1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R
	P–U		P–S	
	Q–T		Q–R	

Now consider team) . Fixtures against teams %, & and * are now fixed in rounds 3. 2 and 4 and, since team ' is unavailable in round 5, team) plays against team ' in round 1 and against team (in round 5. Therefore the missing fixture in round 1 is team (against team * and the complete fixture list can then be completed, as shown below.

1	2	3	4	5
P–Q	R–S	P–T	T–U	P–R
R–T	P–U	Q–S	P–S	S–T
S–U	Q–T	R–U	Q–R	Q–U

Hence team (plays team * in round 1.

25. The diagram shows a quadrilateral divided into four smaller quadrilaterals with a common vertex . The other labelled points divide the sides of the large quadrilateral into three equal parts. The numbers indicate the areas of the corresponding small quadrilaterals. What is the area of the shaded quadrilateral?



A 4 B 5 C 6 D 6.5 E 7

Solution **C**

Label the four vertices of the quadrilateral , , and and join all four vertices to point , as shown. Let the areas of triangles %, (,) and , be G, H, I and F. Since triangles % , (,) and , have the same heights as triangles %, (,) and , but twice the base, their areas are 2G, 2H, 2I and 2F, as shown. Therefore, since the sum of the areas of quadrilaterals ,) and % (is 18 + 10 = 28, we have 2G ¸ 2H ¸ 2I ¸ 2F = 28 and hence G ¸ H ¸ I ¸ F = 14. Hence 8 + area ( ) = 14 and therefore the area of the shaded quadrilateral in the question is 6.