UKMT

United Kingdom
Mathematics Trust

British Mathematical Olympiad

Round 1

Thursday 27 November 1800

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Instructions

1. Time allowed: $2\frac{1}{2}$ hours.
2. Each question in Section A carries 5 marks. Each question in Section B carries 10 marks. Earlier questions tend to be easier; you are advised to concentrate on these problems first.
3. In Section A only answers are required.
4. Use the answer sheet provided for Section A.
5. In Section B full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
6. One complete solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.
7. Write on one side of the paper only. Start each question in Section B on a fresh sheet of paper: *scans of your work will need to be uploaded question by question for marking.*
8. On each sheet of working for Section B, write the number of the question in the top left hand corner and your **Participant ID** and **UKMT centre number**. Do not write you name.
9. The use of rulers, set squares and compasses is allowed, but calculators and protractors are forbidden. *You are strongly encouraged to use geometrical instruments to construct large, accurate diagrams for Section B geometry problems.*
10. At the end of the paper, return to your Section A answer sheet and indicate which Section B questions you have attempted.
11. Please do not discuss the paper on the internet until 8am GMT on Saturday 29 November when the solutions video will be released at https://bmos.ukmt.org.uk
12. **Do not turn over until told to do so.**

Enquiries about the British Mathematical Olympiad should be sent to:

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Section A

The questions in Section A are worth a maximum of five points each.
Only answers are required.
Please use the answer sheet provided.

1. A list of five two-digit positive integers is written in increasing order on a blackboard. Each of the five integers is a multiple of 3, and each digit 0,1,2,3,4,5,6,7,8,9 appears exactly once on the blackboard. In how many ways can this be done? *Note that a two-digit number cannot begin with the digit 0.*

2. For each positive real number x, we define $\{x\}$ to be the greater of x and $1/x$, with $\{1\} = 1$.

Find all positive real numbers y such that

$$5y\{8y\}\{25y\} = 1.$$

3. Determine all pairs (m, n) of positive integers which satisfy the equation

$$n^2 - 6n = m^2 + m - 10.$$

4. There are 1800 penguins waddling towards their favourite restaurant. As the penguins arrive, they are handed tickets numbered in ascending order from 1 to 1800, and told to join the queue. The first penguin starts the queue. For each $n > 1$ the penguin holding ticket number n finds the greatest $m < n$ which divides n and enters the queue directly behind the penguin holding ticket number m. This continues until all 1800 penguins are in the queue.

(a) How many penguins are in front of the penguin with ticket number 2?

(b) What numbers are on the tickets held by the penguins just in front of and just behind the penguin holding ticket 33?

Section B

The questions in Section B are worth a maximum of ten points each.
Full written solutions are required.
Please begin each question on a new sheet of paper.

5. Let Γ be a semicircle with diameter AB. The point C lies on the diameter AB and points E and D lie on the arc BA, with E between B and D. Let the tangents to Γ at D and E meet at F. Suppose that $\angle ACD = \angle ECB$.

Prove that $\angle EFD = \angle ACD + \angle ECB$.

6. Consider sequences $a_1, a_2, a_3, \ldots$ of positive real numbers with $a_1 = 1$ and such that $a_{n+1} + a_n = (a_{n+1} - a_n)^2$ for each positive integer n. How many possible values can a_{1800} take?

7. Ada the ant starts at a point O on a plane. At the start of each minute she chooses North, South, East or West, and marches 1 metre in that direction. At the end of 1800 minutes she finds herself back at O. Let n be the number of possible journeys which she could have made. What is the highest power of 10 which divides n?

