United Kingdom
Mathematics Trust

25th Anniversary Challenge

Solutions 2022

For reasons of space, these solutions are necessarily brief.

There are more in-depth, extended solutions available on the UKMT website, which include some exercises for further investigation:

www.ukmt.org.uk

1. D $(999 - 99 + 9) \div 9 = (900 + 9) \div 9 = 909 \div 9 = 101$.

2. D Remembering that multiplication is always performed in a calculation before addition or subtraction, we note that $3 + 5 \times 4 = 3 + 20 = 23$; $20 - 5 \times 4 = 20 - 20 = 0$; $12 - 5 \times 2 = 12 - 10 = 2$; $3 + 6 \times 4 = 3 + 24 = 27$; and $5 \times 3 - 2 = 15 - 2 = 13$.
So the calculation in option D is incorrect.

3. A 2010 mm = 2.01 m so, of the alternatives given, only a table could be expected to have a length of 2010 mm.

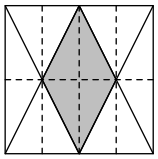
4. E Kylie has four front thumbs and two back thumbs, a total of six. So, between them, ten koalas have 60 thumbs.

5. E An integer will have exactly one factor other than 1 and itself if, and only if, it is the square of a prime. Of the options given, the only such number is 25. Its factors are 1, 5, 25.

6. C The unmarked interior angle on the right of the triangle is $(360 - 324)^\circ = 36^\circ$.
So, by the exterior angle theorem, $x = 100 - 36 = 64$.

7. E Let Amrita's number be n. The information in the question tells us that $(2n + 9) \div 3 - 1 = n$.
Therefore $2n + 9 = 3(n + 1) = 3n + 3$. Hence $n = 9 - 3 = 6$.

8. C Broken lines have been added to the square so that it has been divided up into sixteen congruent triangles, four of which have been shaded.



So the fraction of the area of the square which has been shaded is $\frac{4}{16} = \frac{1}{4}$.

9. D There are 60 seconds in one minute. So the number of seconds in one thousandth of one minute is $60 \div 1000 = 0.06$.

10. E The original pyramid had 8 edges. Cutting off the top corner adds 4 edges, whilst cutting off the other four corners adds 3 extra edges in each case.
So the total number of edges is $8 + 4 + 4 \times 3 = 24$.

11. B The second sign is 143 miles from London and 250 miles from Edinburgh, so it is 393 miles from Edinburgh to London.

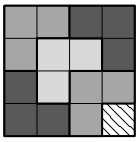
12. D The octopus is able to open 8 jars per minute, so the number opened per hour is $60 \times 8 = 480$.

13. A The mode is the category which contains more than any of the others, so is represented by the largest sector in a pie chart.

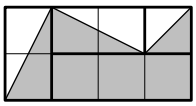
14. A First note that as all of the stamps cost at least 20p, there cannot be more than a total of ten stamps. Therefore, to achieve a total cost ending in 2, there could be two or seven 26p stamps. If there were two such stamps then the cost of the 20p stamps would be $(202 - 52)$p = 150p, but 150 is not divisible by 20. Hence there must be seven 26p stamps to give a total ending in 2. As 7×26p = £1.82, there is only one 20p stamp.

15. C There is no connection between the pair of holes on the left of the card and the pair of holes on the right. There must, however, be such a connection for the display on the card to be as shown. (*It is left to the reader to calculate the order in which the holes must be connected in options A, B, D and E to produce the pattern shown on the front of the card.*)

16. D The diagram shows that it is possible to place five L-shapes on the 4×4 board and, as there is now only one unfilled square, the maximum number of L-shapes which may be placed on the board is five.



17. A In the diagram, the shaded area consists of a rectangle of area 3, a triangle of area $\frac{1}{2}$ and two triangles, each of area 1. So the shaded area is equal to that of five and a half of the eight squares.



Therefore the required fraction is $\dfrac{5\frac{1}{2}}{8} = \dfrac{11}{16}$.

18. D The lengths in minutes of the fifth set and the whole match are 491 and 665 respectively. So the required fraction is $\dfrac{491}{665} = \dfrac{491 \times 3}{665 \times 3} \approx \dfrac{1500}{2000} = \dfrac{3}{4}$.

19. B From the rules in the question, there must be a 2 somewhere in the fifth column. The 2 cannot be placed in the middle right-hand rectangle, since this rectangle already contains a 2; it cannot be placed in the bottom row, since this row already contains a 2. Hence the 2 in the fifth column must go in the cell marked x.

				x	5
				6	q
		1	2		
		3	4		
		4		3	p
2					1

(*It is left to the reader to check that this "mini-sudoku" can be completed as required - although not in a unique fashion.*)

20. B If the Knave of Hearts is telling the truth then the Knave of Clubs is lying, which means that the Knave of Diamonds is telling the truth, but the Knave of Spades is lying. Alternatively, if the Knave of Hearts is lying then the Knave of Clubs is telling the truth, which means that the Knave of Diamonds is lying, but the Knave of Spades is telling the truth. In both cases, we can determine that two of the Knaves are lying, although it is not possible to determine which two they are.

21. D Let the ten consecutive integers be $x - 4, x - 3, x - 2, x - 1, x, x + 1, x + 2, x + 3, x + 4$ and $x + 5$ respectively. The sum of these is $10x + 5$ so $10x + 5 = 5$, that is $x = 0$. Hence the largest of the integers is 5.

22. B Comparing the units column with the tens column shows that there must be a carry from units to tens. This comes from J + M. So J + M = 10 and also, from the tens column, M = C + 1.
As J + M = 10, there cannot be a carry greater than 1 from the tens column so the hundreds column shows that J = 1. Hence M = 10 – J = 9 and C = M – 1 = 8.
Therefore J + M + C = 1 + 9 + 8 = 18.

23. A Let the weights, in kg, of baby, nurse and me be b, n, m respectively. Then $b+m = 78$; $b+n = 69$; $n + m = 137$. Adding all three equations gives $2b + 2n + 2m = 284$, so $b + n + m = 142$.
(*To find the combined weight, it is not necessary to find the individual weights, but baby weighs 5* kg, *nurse weighs 64* kg *and I weigh 73* kg.)

24. B Let p be the total number of pears. Then $12 + \frac{p}{9} = \frac{1}{2}\left(p - \frac{p}{9}\right) = \frac{4p}{9}$.
So $12 = \frac{3p}{9} = \frac{p}{3}$. Therefore $p = 3 \times 12 = 36$.
So the number of pieces of fruit in the first box is $12 + \frac{36}{9} = 12 + 4 = 16$. Hence the number of pieces of fruit in each box is 16.

25. B Let the house number be 'abc'. The sum $a + b + c$ is at most 27, so $a = 4$.
Therefore 'bc' $+ 4 + b + c = 29$ and so 'bc' $+ b + c = 25$. We can rewrite this equation as $11b + 2c = 25$ from which it is clear that $b = 1$ and then $c = 7$.
Therefore the house number is 417 and the product of its digits is 28.