UK JUNIOR MATHEMATICAL CHALLENGE

THURSDAY 30th APRIL 2009

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

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SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **B** $9002 - 2002 = 7000$ so $9002 - 2009 = 7000 - 7 = 6993$.

2. **B** Each of faces 1, 4 and 5 has four axes of symmetry, whilst each of faces 2, 3 and 6 has two axes of symmetry only.

3. **D** The values of the left-hand sides of the expressions are 0, 16, 28, 36 and 40 respectively.

4. **E** Each of points A, B, C and D is 1 unit from the origin, but the point (1, 1) is at a distance $\sqrt{2}$ units from the origin.

5. **D** The problem may be solved by dividing each of the alternatives in turn by 7, but the prime factorisation of 1001, i.e. $1001 = 7 \times 11 \times 13$, leads to the conclusion that 111 111, which is 111×1001, is a multiple of 7.

6. **B** Triangle ABM has base 3 units and height 3 units, so its area is $\frac{1}{2} \times 3 \times 3$ units^2, that is $4\frac{1}{2}$ units^2.

7. **C** The time difference is 12 hours and 12 minutes, that is 732 minutes.

8. **E** Removing tile A or tile B or tile D has the effect of reducing the perimeter by a distance equal to twice the side of one tile, whilst removing tile C increases the perimeter by that same distance. Removing tile E, however, leaves the length of the perimeter unchanged.

9. **A** $\frac{20}{11} = 1\frac{9}{11} = 1.818181\ldots$, so only two different digits appear.

10. **B** The triangle in the centre of the diagram is equilateral since each of its sides is equal in length to the side of one of the squares. The sum of the angles at a point is 360°, so $x = 360 - (90 + 90 + 60) = 120$.

11. **C** The first thirteen terms of the sequence are $-3, 0, 2, -1, 1, 2, 2, 5, 9, 16, 30, 55, 101, \ldots$.

12. **A** The increase in Gill's weight is 45 kg, which is 9 times her weight in 1988.
So the percentage increase in weight is 900%.
(*The problem refers to* Q14 *in the very first Schools Mathematical Challenge − the forerunner of the current Junior and Intermediate Mathematical Challenges – in* 1988. *This was '*Weighing the baby at the clinic was a problem. The baby would not keep still and caused the scales to wobble. So I held the baby and stood on the scales while the nurse read off* 78 kg. *Then the nurse held the baby while I read off* 69 kg. *Finally I held the nurse while the baby read off* 137 kg. *What is the combined weight of all three* (*in* kg)?
A 142 B 147 C 206 D 215 E 284.')

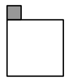
13. **D** Let the ten consecutive integers be $x - 4, x - 3, x - 2, x - 1, x, x + 1, x + 2, x + 3, x + 4$ and $x + 5$ respectively. The sum of these is $10x + 5$ so $10x + 5 = 5$, that is $x = 0$. Hence the largest of the integers is 5.

14. **E** The sum of Karen's two marks was 78×2, that is 156. So her mark for Mathemagics was $156 - 72$, that is 84.

15. **E** If Matt takes 12 jellybeans then he will have taken at least one of each flavour unless he takes all 8 watermelon jellybeans and either all 4 vanilla jellybeans or all 4 butter popcorn jellybeans. In this case the 4 remaining jellybeans will all be of the flavour he has yet to take, so taking one more jellybean ensures that he will have taken at least one of each flavour.

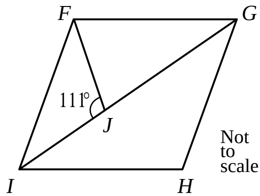
16. **D** 20% of the 80% is 16% of the kettle's capacity. Therefore the volume of water left in the kettle after Keith has poured out 20% of the original amount is 64% of the kettle's capacity. So when full, the kettle holds $\frac{1152}{64} \times 100$ ml, that is 1800 ml.

17. **A** The tiling pattern may be considered to be a tessellation by the shape shown, so the required ratio is 1:1.

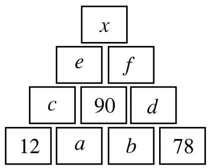


18. **B** The lowest common multiple of 2, 3, 4, 5 and 6 is required. Of these numbers, 2, 3 and 5 are prime whilst $4 = 2^2$ and $6 = 2 \times 3$. So their lowest common multiple is $2^2 \times 3 \times 5$, that is 60.

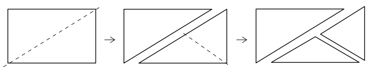
19. **A** Adjacent angles on a straight line add up to 180°, so $\angle GJF = 180° - 111° = 69°$. In triangle FGJ, $GJ = GF$ so $\angle GFJ = \angle GJF$. Therefore $\angle FGJ = (180 - 2 \times 69)° = 42°$. As $FGHI$ is a rhombus, $FG = FI$ and therefore $\angle GIF = \angle FGI = 42°$. Finally, from triangle FJI, $\angle JFI = (180 - 111 - 42)° = 27°$.

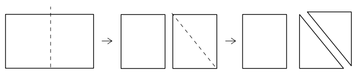


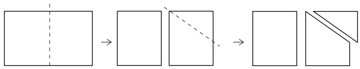
20. **D** Let the numbers in the boxes be as shown in the diagram. Then $b = 90 - a$; $c = 12 + a$; $d = b + 78 = 168 - a$. Also, $e = 90 + c = 102 + a$; $f = 90 + d = 258 - a$. So $x = e + f = 102 + a + 258 - a = 360$.

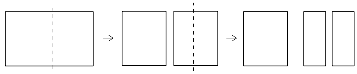


21. **E** The diagrams below show how the total number of edges of the resulting three pieces may be 9, 10, 11 or 12. However, 12 is the maximum value of the total number of edges since the original number of edges is four and any subsequent cut adds a maximum of four edges (by dividing two existing edges and adding the new 'cuts').

9: 

10: 

11: 

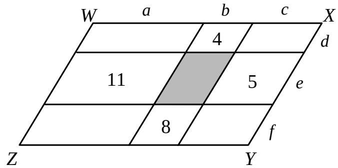
12: 

22. **D** In order to reach a 9 in three steps, the first zero must be one of the three adjacent to the 2 and the second zero must be one of the five adjacent to a 9. The table shows the number of such routes to that point.
So the total number of different routes is 25.

	1	2	**4**
1	1	2	**5**
2	2	1	**3**
4	**5**	**3**	**1**

23. **C** Let the value of a green note and the value of a blue note be g zogs and b zogs respectively. Then $3g + 8b = 46$ and $8g + 3b = 31$. Adding these two equations gives $11g + 11b = 77$, so $b + g = 7$.
Therefore $3g + 3b = 21$. Subtracting this equation from the original equations in turn gives $5b = 25$ and $5g = 10$ respectively. So $b = 5$, $g = 2$ and $2g + 3b = 19$.

24. **C** Let the lengths a, b, c, d, e, f be as shown in the diagram. Then the sum of the perimeters of the four labelled parallelograms is
$2(a+e)+2(b+d)+2(b+f)+2(c+e)$
$= 2(a+b+c+d+e+f)+2(b+e)$
= perimeter of $WXYZ$ + perimeter of shaded parallelogram.



So the perimeter of the shaded parallelogram is $\left((11+8+4+5)-21\right)\text{cm} = 7\,\text{cm}$.

25. **B** Let the number of boys in Miss Quaffley's class be b and the number of girls be g. Then the number of teddy bears is $\frac{1}{3}(b+g)$. Also, in total, the boys took out $12b$ library books last term and the girls took out $17g$ books. The total number of books taken out by the bears was $9 \times \frac{1}{3}(b+g)$ that is $3(b+g)$.
So $12b + 17g + 3(b+g) = 305$, that is $15b + 20g = 305$, that is $3b + 4g = 61$.
Clearly, b and g are positive integers. The positive integer solutions of the equation $3b + 4g = 61$ are $b = 3, g = 13$; $b = 7, g = 10$; $b = 11, g = 7$; $b = 15, g = 4$; $b = 19, g = 1$.
However, there is one further condition: the number of teddy bears, that is $\frac{1}{3}(b+g)$, is also a positive integer and of the five pairs of solutions above, this condition is satisfied only by $b = 11, g = 7$.
Check: the 11 boys take out 132 books, the 7 girls take out 119 books and the 6 teddy bears take out 54 books, giving a total of 305 books.
(*The equation* $3b + 4g = 61$ *in which b and g both represent positive integers is an example of a Diophantine equation*.)