UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 6th FEBRUARY 2003

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the IMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. **E** $3 \div \frac{1}{2} = 3 \times 2 = 6$.

2. **D** The words are MILE and YARD.

3. **A** $643566 \div 2786 = 231$ so $643566 \div 27.86 = 231 \times 100 = 23100$.

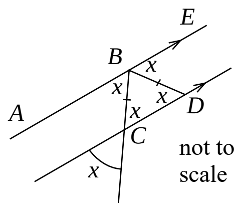
4. **C** One mile is approximately equal to 1.6 kilometres so 120 miles are approximately equal to 200 kilometres.

5. **E** $(2000 + 3)^2 = 2000^2 + 2 \times 2000 \times 3 + 3^2 = 4\,000\,000 + 12\,000 + 9 = 4\,012\,009$.

6. **D** Let the number thought of be x. Then the final number is $4(2x + 3) - 5 - x = 7x + 7 = 7(x + 1)$.

7. **C** The transfer fee for a total of 12 players would be three times that for 4 players.

8. **D** Angle $BCD = x$ (vertically opposite angles);
Angle BDC = angle BCD (base angles of isosceles triangle);
Angle ABC = angle BCD (alternate angles);
Angle EBD = angle BDC (alternate angles).

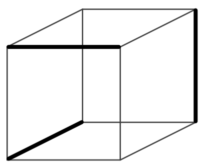


9. **E** 60 million × 70 kg = 4 200 million kg = 4.2 million tonnes, since 1 tonne = 1000 kg.

10. **C** The digits will next be all the same at 11.11, i.e. in 5 hours and 16 minutes time.

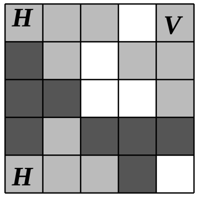
11. **B** The fractions exceed $\frac{1}{2}$ by $\frac{1}{14}, \frac{1}{8}, \frac{1}{4}, \frac{3}{22}$ and $\frac{3}{26}$ respectively i.e. by $\frac{3}{42}, \frac{3}{24}, \frac{3}{12}, \frac{3}{22}$ and $\frac{3}{26}$. Thus the fraction in the middle will be that which exceeds $\frac{1}{2}$ by $\frac{3}{24}$.
[*Alternatively, when the fractions are written as decimals, correct to 3 decimal places, their values are 0.571, 0.625, 0.75, 0.636 and 0.615 respectively.*]

12. **B** Each edge of the cube borders two faces. As there are 6 faces, a minimum of three black edges will be required. The diagram shows that the required condition may indeed be satisfied with three black edges.

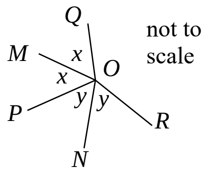


13. **C** The diagram shows that it is possible to fit five T shapes in the square. In order to fit six T shapes into the square, exactly one of the 25 squares would be left uncovered; hence at least three corner squares must be covered.

We now label a corner square H or V if it is covered by a T shape which has the top part of the T horizontal or vertical respectively. If all four corner squares are covered then there must be at least two cases of an H corner with an adjacent V corner. Each such combination produces a non-corner square which cannot be covered e.g. the second square from the right on the top row of the diagram. If only three corner squares are covered, there must again be at least one H corner with an adjacent V corner and therefore a non-corner square uncovered, as well as the uncovered fourth corner. In both cases, at least two squares are uncovered, which means that it is impossible to fit six T shapes into the square.



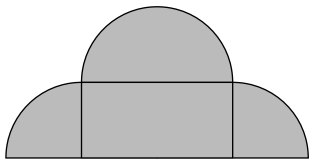
14. **A** As OQ is the reflection of OP in OM, $\angle QOM = \angle POM$; similarly, $\angle RON = \angle PON$. Hence reflex $\angle QOR = 2 \times \angle MON = 260°$. Therefore $\angle QOR = 360° - 260° = 100°$.



15. **E** As each interior angle of the polygon is a whole number of degrees, the same must apply to each exterior angle. The sum of the exterior angles of a polygon is 360° and so the greatest number of sides will be that of a 360-sided polygon in which each interior angle is 179°, thus making each exterior angle 1°.

16. **C** Minnie's average speed this year is $\frac{5}{4}$ of last year's value. Hence her time this year will be $\frac{4}{5}$, i.e. 80%, of last year's time.

17. **A** The shaded area may be divided into a 2×1 rectangle plus a semicircle and two quarter circles, all of radius 1. Hence the total area is that of the rectangle plus that of a circle of radius 1 i.e. $2 + \pi$.



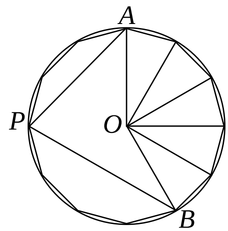
18. **E** All palindromic dates this century will fall in February as they will be of the form *ab* 02 20*ba*. The palindromic dates next century will fall in December as they will be of the form *cd* 12 21*dc* and the first of these will be the 10th of December 2101 i.e. 10 12 2101.

19. **B** The area of the pentagon is that of a rectangle of length b and breadth a plus that of a triangle of base b and height $(c - a)$ i.e. $ab + \frac{1}{2}b(c - a) = \frac{1}{2}b(2a + c - a) = \frac{1}{2}b(a + c)$.
[The area may also be considered to be the sum of the areas of two trapezia which have parallel sides a and c and whose heights have total length b.]

20. **D** We are given that $2n + e = 8$ and $t + e + n = 10$. Subtracting the first equation from the second gives $t - n = 2$. As n cannot equal 1, the minimum value of t is 4 but this gives $n = 2$, $e = 4$ which is impossible. If $t = 5$ then $n = 3$ and $e = 2$, which is allowed. If $t > 5$ then $n > 3$ and e is not a positive whole number, so 5 is the only possible value of t.

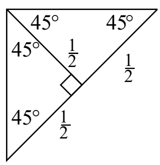
21. **D** Let the radius of R's track be r and let the radius of the first semicircle of P's track be p; then the radius of the second semicircle of this track is $r - p$. The total length of P's track is $\pi p + \pi(r - p) = \pi r$, the same as the length of R's track. By a similar argument, the length of Q's track is also πr, so all three runners finish at the same time.

22. **C** Each side of the dodecagon subtends an angle of 30° at the centre of the circumcircle of the figure (the circle which passes through all 12 of its vertices). Thus $\angle AOB = 150°$ and, as the angle subtended by an arc at the centre of a circle is twice the angle subtended by that arc at a point on the circumference, $\angle APB = 75°$.



23. **A** It takes ab man-hours to paint c square metres of the bridge and hence $\frac{ab}{c}$ man-hours to paint 1 square metre. So d men will take $\frac{ab}{cd}$ hours to paint 1 square metre and $\frac{abe}{cd}$ hours to paint e square metres.

24. **B** The figure shows the top left-hand corner of the complete diagram and we see by symmetry that the perpendicular from the corner to the short side of the rectangle has length $\frac{1}{2}$. Thus the diagonal of the square may be divided into three sections of length $\frac{1}{2}$, x and $\frac{1}{2}$ respectively.



The length of this diagonal $= \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$, so $x = 10\sqrt{2} - 1$.

25. **A** Let the widescreen width and traditional width be w and W respectively. Then the respective heights are $\frac{9w}{16}$ and $\frac{3W}{4}$. As the areas are equal:
$w \times \frac{9w}{16} = W \times \frac{3W}{4}$ i.e. $w^2 = \frac{4}{3}W^2$. Hence $w : W = 2 : \sqrt{3}$.