United Kingdom Mathematics Trust

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Solutions

1. D It is known that any diagram with at most two points where an odd number of lines meet can be drawn without lifting your pencil off the page and without drawing along the same line twice. Any diagram with more than two such points cannot be drawn in this way. Of the options given, only diagram D has more than two such points. Hence the diagram which cannot be drawn is D.

2. B Since $2 + 0 + 1 + 9 = 12$ and by inserting brackets only + or − signs are possible between the numbers, no value greater than 12 may be obtained. Also, $2 - (0 - 1 - 9) = 2 - 0 + 1 + 9 = 12$ and hence a result of 12 is possible. Therefore the largest possible value that can be obtained is 12.

3. C Kerry uses the digit 0 five times and hence the value of n is smaller than 60. She uses the digit 9 six times and hence the value of n is at least 59. Therefore the value of n is 59.

4. D The largest grey square is a quarter of the large square. The smaller grey squares are each one ninth of the size of the largest grey square. Hence the fraction of the large square which is shaded is $\frac{1}{4} + \frac{7}{9} \times \frac{1}{4} = \frac{1}{4} \times (1 + \frac{7}{9}) = \frac{1}{4} \times \frac{16}{9} = \frac{4}{9}$

5. A The information in the question tells us that Lotar finished before Manfred who finished before Jan who finished before Victor. Since Eddy also finished before Victor, it was Victor who finished last of these five runners.

6. D Each of the five friends gave a cake to the four other people. Therefore the number of cakes given away and then eaten was $4 \times 5 = 20$. Since this decreased the total number of cakes they had by half, the total number of cakes they had at the start was $2 \times 20 = 40$.

7. A The required sum can be written as shown below, with a, b and c as the missing digits:

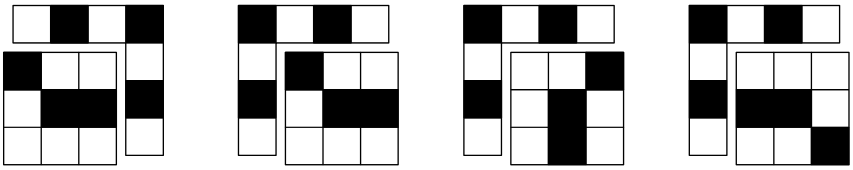
$$\begin{array}{rrrrr} & 1 & 2 & 4 & 3 \\ & 2 & 1 & a & 7 \\ + & b & c & 2 & 6 \\ \hline 1 & 0 & 1 & 2 & 6 \end{array}$$

The sum of the digits in the units column is 16 and hence there is a carry of 1 to the tens column. Therefore, when we consider the tens column, we have $4 + a + 2 + 1 = 2$ or $2 + 10$. Hence $7 + a = 2$ or 12 and, since a is a positive single-digit integer, $a = 5$ and there is a carry of 1 to the hundreds column. Similarly, when we consider the hundreds column, we have $2 + 1 + c + 1 = 1$ or 11 and hence $c = 7$ and there is a carry of 1 to the thousands column. Finally, when we consider the thousands and ten thousands columns, we have $1 + 2 + b + 1 = 10$ and hence $b = 6$. Therefore the missing digits are 5, 6 and 7.

8. C Let the number of apples Andrew had be $6n$. When Boris divided the same number of apples into five piles, each pile contained two more apples than each of Andrew's piles. Therefore $6n = 5(n + 2)$ and hence $6n = 5n + 10$. This has solution $n = 10$. Therefore the number of apples Andrew had was $6 \times 10 = 60$.

9. **B** Since $PQ = QS$, triangle PSQ is isosceles and hence $\angle PSQ = 20°$. Since the angles in a triangle add to $180°$, we have $20° + 20° + \angle SQP = 180°$ and hence $\angle SQP = 140°$. Since $PQ = PR$, triangle PRQ is isosceles and hence $\angle PRQ = \angle RQP$. Also $\angle PRQ + \angle RQP + 20° = 180°$ and hence $\angle RQP = 80°$. Since $\angle RQS = \angle SQP - \angle RQP$, the size of $\angle RQS$ is $140° - 80° = 60°$.

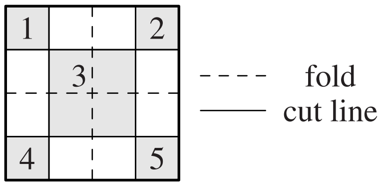
10. **E** When the two given pieces are joined together, any resulting square must have on its outside one row and one column, each of which have alternating black and white squares. Therefore tile E cannot be made. The diagrams below show how the tiles in options A, B, C and D can be made by combining the given pieces, confirming E as the only tile which cannot be made.



11. **B** Since Dora shook hands four times, she shook hands with all the other four people. Hence, since Alan only shook hands once, it was with Dora. Since Claire shook hands three times and did not shake hands with Alan, she shook hands with Bella, Dora and Erik. Hence, since Bella only shook hands twice, it was with Dora and Claire. Therefore Erik shook hands twice (with Dora and Claire).

12. **C** Since Jane had a success rate of 55% after her first 20 shots, the number of times she had scored out of 20 was $0.55 \times 20 = 11$. Similarly, since her success rate had increased to 56% after 5 more shots, the number of times she had scored out of 25 was $0.56 \times 25 = 14$. Hence the number of shots she scored out of the last five was $14 - 11 = 3$.

13. **C** When Cathie cut the paper as described in the question, her cuts divided the original paper as shown in the diagram. It can then be seen that the five pieces shaded are squares.



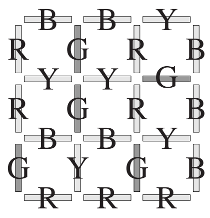
14. **D** Michael has 24 pets. Since $\frac{1}{8}$ of them are dogs, he has 3 dogs. Since $\frac{3}{4}$ are not cows, $\frac{1}{4}$ of them are cows and hence he has 6 cows. Similarly, since $\frac{2}{3}$ are not cats, $\frac{1}{3}$ of them are cats and hence he has 8 cats. Therefore, the number of kangaroos he has is $24 - 3 - 6 - 8 = 7$.

15. **B** Since the length of five identical rectangles is 10 cm, the length of one rectangle is 2 cm. Similarly, since the height of four rectangles is 6 cm, the height of one rectangle is 1.5 cm. Therefore the total area of the 14 rectangles is $14 \times (2 \times 1.5)\,\text{cm}^2 = 42\,\text{cm}^2$. Hence the area of the shaded region is equal to $(42 - \frac{1}{2} \times 10 \times 6)\,\text{cm}^2 = (42 - 30)\,\text{cm}^2 = 12\,\text{cm}^2$.

16. **C** First note that the factorisation of 2331 into prime numbers is $2331 = 3 \times 3 \times 7 \times 37$. Note also that $3 \times 37 = 111$. Since Claire's three-digit integer has all its digits different, and both $3 \times 111 = 333$ and $7 \times 111 = 777$ have repeated digits, the factors of Claire's integer do not include both 3 and 37. However, since $3 \times 3 \times 7 = 63$, which is not a three-digit integer, the factors of Claire's integer must include 37. Therefore Claire's integer is $37 \times 7 = 259$ and hence the number of pieces of paper Peter picked is $2331 \div 259 = 9$.

17. **C** Let the original heights of the first and second candles be x and y respectively. Since the first candle lasts six hours, after three hours its height is $\frac{1}{2}x$. Similarly, since the second candle lasts eight hours, after three hours its height is $\frac{5}{8}y$. After three hours the two candles have the same height and hence $\frac{1}{2}x = \frac{5}{8}y$. Therefore $\frac{x}{y} = \frac{5}{4}$ and hence the ratio of their original heights is $5 : 4$.

18. C Since each 1×1 square has four sides of different colours, there is a green stick along the side of each square. Also, since any stick is part of at most two squares four green sticks could only contribute to at most eight squares. Therefore at least five green sticks are needed. The diagram on the right shows that such an arrangement is possible with five green sticks. Hence the smallest number of green sticks she could use is five.



19. B Since the integer 7 is joined by a diameter to the integer 23, we can deduce that there are $23 - 7 + 1 = 15$ integers between them on each side. Therefore there are $2 \times 15 + 2 = 32$ integers in total round the circle. Hence the value of n is 32.

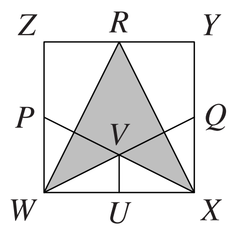
20. B Liam spent $50 \times £1 = £50$ buying the bottles. After selling 40 bottles he had $£(50 + 10) = £60$. Therefore he sold each bottle for $£60 \div 40 = £1.50$. Hence, after selling all 50 bottles, he then had $50 \times £1.50 = £75$.

21. A Since no two circles that are joined directly are painted the same colour and circles 2, 5 and 6 are joined to each other, they are all painted different colours. Similarly circles 2, 6 and 8 join to each other and hence are painted different colours. Therefore circles 5 and 8 must have been painted the same colour. It is easy to check that, given any other pair of circles in the diagram, it is possible for them to be coloured differently.

22. C The original ratio of the amounts of money in Ria's and Flora's savings accounts was $5 : 3 = 25 : 15$. After Ria withdrew 160 euros, the ratio changed to $3 : 5 = 9 : 15$. Since Flora's savings have not changed, the 160 euros Ria withdrew represented $\frac{(25-9)}{25} = \frac{16}{25}$ of her original savings. Therefore 10 euros represented $\frac{1}{25}$ of her original savings and hence she originally had 250 euros.

23. E Let the largest number of teams that can enter the tournament be n. Hence there would be $3n$ players in total. Each player in a team will play a game against every player from all other teams and hence the total number of games played is $\frac{3n \times (3n-3)}{2} = \frac{9n(n-1)}{2}$. Since no more than 250 games can be played, we have $\frac{9n(n-1)}{2} \leq 250$. Therefore $n(n-1) \leq \frac{500}{9} = 55\frac{5}{9}$. Since $8 \times 7 = 56 > 55\frac{5}{9}$, we have $n < 8$. Also, since $7 \times 6 = 42 < 55\frac{5}{9}$, $n = 7$ is a possible solution. Therefore the largest number of teams that can enter the tournament is 7.

24. E Label the intersection of WQ and XP as V and the midpoint of WX as U. Let the side-length of the square be 1 unit. The area of triangle WXR is $\frac{1}{2} \times 1 \times 1$ units^2 $= \frac{1}{2}$ units^2. Consider triangle WXP and triangle UXV. These two triangles have the same angles and hence are similar. Since UX is half of WX, it follows that VU is half of PW and hence has length $\frac{1}{4}$ unit. Therefore the area of triangle WXV is $\left(\frac{1}{2} \times 1 \times \frac{1}{4}\right)$ units^2 $= \frac{1}{8}$ units^2. Hence the shaded area is $(\frac{1}{2} - \frac{1}{8})$ units^2 $= \frac{3}{8}$ units^2. Therefore the fraction of the square that is shaded is $\frac{3}{8}$.



25. D Let n be the total number of passengers in the middle two carriages. These are the ninth and tenth carriages and so, since there are 199 passengers in any block of five adjacent carriages, the total number of passengers in the sixth, seventh and eighth carriages is $199 - n$. There are 199 passengers in total in carriages 1 to 5 inclusive, in carriages 9 to 13 inclusive and in carriages 14 to 18 inclusive. Since there are 700 passengers in total on the train, we have $199 + 199 - n + 199 + 199 = 700$. Therefore $796 - n = 700$ and hence $n = 96$. Therefore there are 96 passengers in total in the middle two carriages of the train.