

United Kingdom  
Mathematics Trust

# INTERMEDIATE MATHEMATICAL OLYMPIAD

## CAYLEY PAPER

Thursday 21 March 2019

Organised by the United Kingdom Mathematics Trust



*England & Wales: Year 9 or below*  
*Scotland: S2 or below*  
*Northern Ireland: Year 10 or below*

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

*UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT*

☎ 0113 343 2339

enquiry@ukmt.org.uk

www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do not hand in rough work.*

1. Each of Alice and Beatrice has their birthday on the same day.

In 8 years' time, Alice will be twice as old as Beatrice. Ten years ago, the sum of their ages was 21.

How old is Alice now?

2. In the addition shown, each of the letters  $D$ ,  $O$ ,  $G$ ,  $C$ ,  $A$  and  $T$  represents a different digit.

$$\begin{array}{r} DOG \\ + CAT \\ \hline 1000 \end{array}$$

What is the value of  $D + O + G + C + A + T$ ?

3. The triangle  $ABC$  is isosceles with  $AB = BC$ . The point  $D$  is a point on  $BC$ , between  $B$  and  $C$ , so that  $AC = AD = BD$ .

What is the size of angle  $ABC$ ?

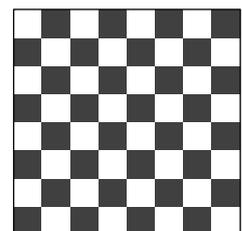
4. Arrange the digits 1, 2, 3, 4, 5, 6, 7, 8 to form two 4-digit integers whose difference is as small as possible.

Explain clearly why your arrangement achieves the smallest possible difference.

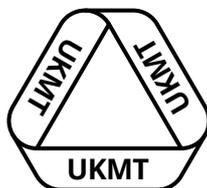
5. Howard chooses  $n$  different numbers from the list 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, so that no two of his choices add up to a square.

What is the largest possible value of  $n$ ?

6. A chessboard is formed from an  $8 \times 8$  grid of alternating black and white squares, as shown. The side of each small square is 1 cm.



What is the largest possible radius of a circle that can be drawn on the board in such a way that the circumference is entirely on white squares or corners?



United Kingdom  
Mathematics Trust

# INTERMEDIATE MATHEMATICAL OLYMPIAD

## HAMILTON PAPER

**Thursday 21 March 2019**

Organised by the United Kingdom Mathematics Trust



*England & Wales: Year 10*  
*Scotland: S3*  
*Northern Ireland: Year 11*

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

*UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT*

☎ 0113 343 2339

enquiry@ukmt.org.uk

www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do not hand in rough work.*

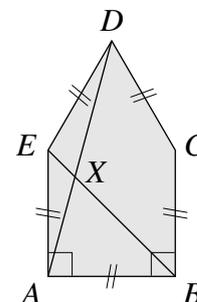
1. A number of couples met and each person shook hands with everyone else present, but not with themselves or their partners.

There were 31 000 handshakes altogether.

How many couples were there?

2. The diagram shows a pentagon  $ABCDE$  in which all sides are equal in length and two adjacent interior angles are  $90^\circ$ . The point  $X$  is the point of intersection of  $AD$  and  $BE$ .

Prove that  $DX = BX$ .



3. A  $4\text{ cm} \times 4\text{ cm}$  square is split into four rectangular regions using two line segments parallel to the sides.

How many ways are there to do this so that each region has an area equal to an integer number of square centimetres?

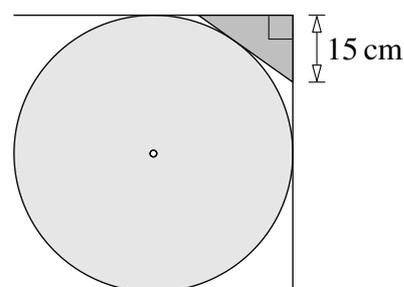
4. Each of  $A$  and  $B$  is a four-digit palindromic integer,  $C$  is a three-digit palindromic integer, and  $A - B = C$ .

What are the possible values of  $C$ ?

[A palindromic integer reads the same 'forwards' and 'backwards'.]

5. The area of the right-angled triangle in the diagram alongside is  $60\text{ cm}^2$ . The triangle touches the circle, and one side of the triangle has length  $15\text{ cm}$ , as shown.

What is the radius of the circle?



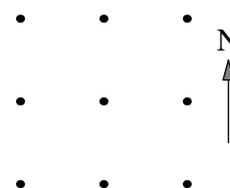
6. Nine dots are arranged in the  $2 \times 2$  square grid shown. The arrow points north.

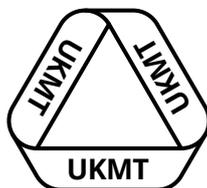
Harry and Victoria take it in turns to draw a unit line segment to join two dots in the grid.

Harry is only allowed to draw an east-west line segment, and Victoria is only allowed to draw a north-south line segment. Harry goes first.

A point is scored when a player draws a line segment that completes a  $1 \times 1$  square on the grid.

Can either player force a win, no matter how the other person plays?





United Kingdom  
Mathematics Trust

# INTERMEDIATE MATHEMATICAL OLYMPIAD

## MACLAURIN PAPER

**Thursday 21 March 2019**

Organised by the United Kingdom Mathematics Trust



*England & Wales: Year 11*

*Scotland: S4*

*Northern Ireland: Year 12*

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

*UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT*

☎ 0113 343 2339

enquiry@ukmt.org.uk

www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
- ◇ *Your answers should be fully simplified, and exact. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but not decimal approximations.*
- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
- ◇ *Just stating an answer, even a correct one, will earn you very few marks.*
- ◇ *Incomplete or poorly presented solutions will not receive full marks.*
- ◇ *Do not hand in rough work.*

1. A train leaves K for L at 09:30 while another train leaves L for K at 10:00. The first train arrives in L 40 minutes after the trains pass each other. The second train arrives in K 1 hour and 40 minutes after the trains pass.

Each train travels at a constant speed.

At what time did the trains pass each other?

2. A right-angled triangle has area  $150 \text{ cm}^2$  and the length of its perimeter is 60 cm.

What are the lengths of its sides?

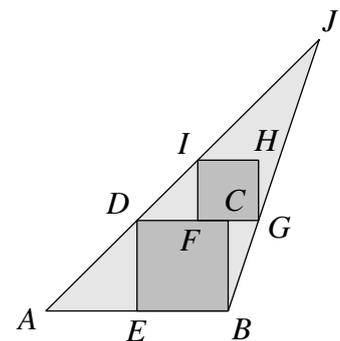
3. Two numbers are such that the sum of their reciprocals is equal to 1. Each of these numbers is then reduced by 1 to give two new numbers.

Prove that these two new numbers are reciprocals of each other.

[The reciprocal of a non-zero number  $x$  is the number  $\frac{1}{x}$ .]

4. The diagram shows the two squares  $BCDE$  and  $FGHI$  inside the triangle  $ABJ$ , where  $E$  is the midpoint of  $AB$  and  $C$  is the midpoint of  $FG$ .

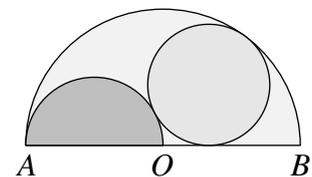
What is the ratio of the area of the square  $BCDE$  to the area of the triangle  $ABJ$ ?



5. A semicircle of radius 1 is drawn inside a semicircle of radius 2, as shown in the diagram, where  $OA = OB = 2$ .

A circle is drawn so that it touches each of the semicircles and their common diameter, as shown.

What is the radius of the circle?



6. A tiling of an  $n \times n$  square grid is formed using  $4 \times 1$  tiles.

What are the possible values of  $n$ ?

[A tiling has no gaps or overlaps, and no tile goes outside the region being tiled.]