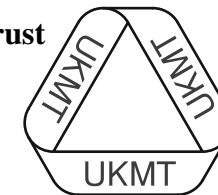


The United Kingdom Mathematics Trust



**Intermediate Mathematical Olympiad and Kangaroo  
(IMOK)**

**Olympiad Hamilton Paper**

Thursday 14th March 2013

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

**READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**  
Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.  
You may wish to work in rough first, then set out your final solution with clear explanations and proofs.  
**Do not hand in rough work.**
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.  
Do not hurry, but spend time working carefully on one question before attempting another.  
Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

**DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!**

The United Kingdom Mathematics Trust is a Registered Charity.

*Enquiries should be sent to: Maths Challenges Office,*

*School of Mathematics, University of Leeds, Leeds, LS2 9JT.*

*(Tel. 0113 343 2339)*

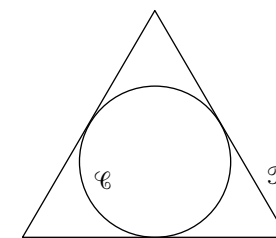
*<http://www.ukmt.org.uk>*

- *Do not hurry, but spend time working carefully on one question before attempting another.*
- *Try to finish whole questions even if you cannot do many.*
- *You will have done well if you hand in full solutions to two or more questions.*
- *Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as  $\pi$ , fractions, or square roots, if appropriate, but NOT decimal approximations.*
- *Give full written solutions, including mathematical reasons as to why your method is correct.*
- *Just stating an answer, even a correct one, will earn you very few marks.*
- *Incomplete or poorly presented solutions will not receive full marks.*
- **Do not hand in rough work.**

1. If  $xy = 10$  and  $(x + 1)(y + 1) = 20$ , what is the value of  $(x + 2)(y + 2)$ ?

2. The sides of an equilateral triangle  $\mathcal{T}$  are three tangents of a circle  $\mathcal{C}$ , as shown. Prove that

$$\frac{\text{area of } \mathcal{C}}{\text{area of } \mathcal{T}} = \frac{\text{circumference of } \mathcal{C}}{\text{perimeter length of } \mathcal{T}}.$$



3. Pablo plans to take several unit cubes and arrange them to form a larger cube. He will then paint some of the faces of the larger cube. When the paint has dried, he will split the larger cube into unit cubes again.

Suppose that Pablo wants exactly 150 of the unit cubes to have no paint on them at all. How many faces of the larger cube should he paint?

4. The vertices of a square have coordinates  $(p, 0)$ ,  $(a, b)$ ,  $(c, d)$  and  $(0, q)$ , where  $a, b, c, d, p$  and  $q$  are all positive.

Prove that  $p + q = \frac{1}{3}(a + b + c + d)$ .

5. When Anne entered the room, the mean age increased by 4 years. When Beth then entered the room, the mean age increased by a further 3 years. Anne and Beth were the same age.

How many people were in the room before Anne entered?

6. Two snails slither at the same speed around the perimeter of triangle  $XYZ$ , in which  $\angle Y$  is a right angle. They start together at  $X$ , one travelling clockwise, the other anticlockwise, until they meet at the point  $P$  on  $YZ$ .

Prove that

$$\frac{2}{XY} + \frac{1}{YP} = \frac{1}{PZ}.$$