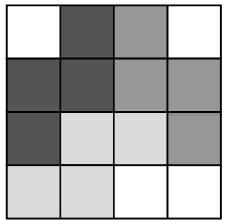
Solutions to the European Kangaroo Grey Paper

1. **D** $\frac{3333}{101} = 3 \times \frac{1111}{101} = 33$ and $\frac{6666}{303} = 6 \times \frac{1}{3} \times \frac{1111}{101} = 22$.

Therefore the original sum is 55.

2. **C** Ann is cutting out shapes made up of four cells from an original square of 16 cells. It is possible to cut out three shapes in a number of different ways, one of which is shown in the diagram. However, it is not possible to cut out four such shapes. To cut out four such shapes, Anne would need to use all 16 cells. Consider the bottom left corner cell. The only possibilities for this cell to be used are in the lightest shaded shape as shown or in the darkest shaded shape moved down one cell. In the first case, the bottom right corner cell could not be used while in the second case, the top left corner cell could not be used. Hence it is impossible to use all 16 cells. So the largest number of shapes Anne can cut out is three and so the smallest number of cells she can leave unused is $16 - 3 \times 4 = 4$.



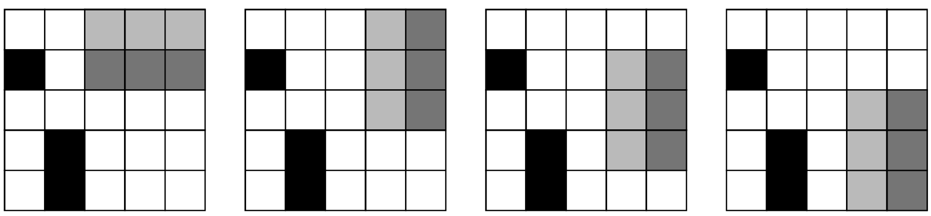
3. **E** No 1-digit number has a digital product of 24. However, $24 = 3 \times 8$ and $24 = 4 \times 6$ and these are the only ways to write 24 as the product of two single digit numbers. Hence there are precisely four 2-digit numbers (38, 83, 46 and 64) with digital product 24. The smallest of these is 38, which has a digital sum of 11.

4. **E** The total number of children in five families is equal to five times the mean. Of the options given, only $2.5 \times 5 = 12.5$ does not give a whole number. Therefore, the mean number of children cannot be 2.5.

5. **B** Nicky's speed is $\frac{9}{8}$ of Rachel's speed so, for each lap of the fountain Rachel completes, Nicky gains $\frac{1}{8}$ of a lap. To catch Rachel, Nicky has to gain $\frac{1}{2}$ a lap. This will take $\frac{1}{2} \div \frac{1}{8} = 4$ laps.

6. **C** We observe that x divides both 14 and 35, so $x = 1$ or 7. If $x = 1$, then from $xz = 35$, we deduce that $z = 35$. But this is impossible as y is an integer and $yz = 10$. Therefore $x = 7$. Hence as $xy = 14$, we have $y = 2$ and as $xz = 35$, $z = 5$. So $x + y + z = 7 + 2 + 5 = 14$.

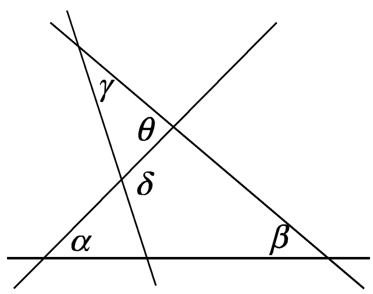
7. **E** The 3×1 ship can be placed in two positions horizontally and six positions vertically as shown making a total of eight positions.



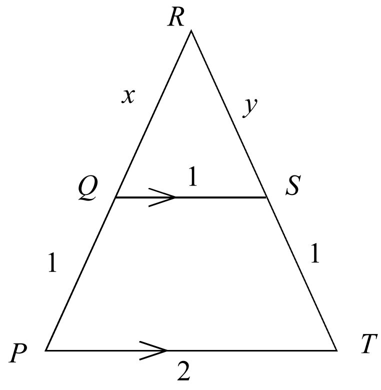
8. **E** Let θ be the angle as shown in the diagram.

As the exterior angle of a triangle is equal to the sum of the two interior opposite angles, we have $\theta = \alpha + \beta$ and $\delta = \gamma + \theta$.

This gives $\delta = \alpha + \beta + \gamma = 55° + 40° + 35° = 130°$.

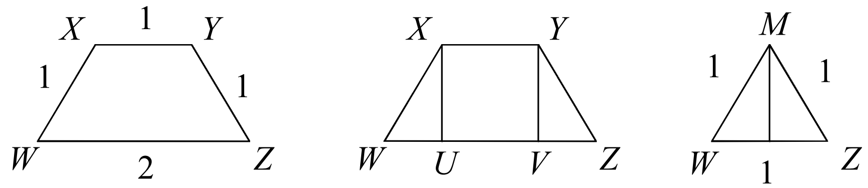


9. B The situation as described must refer to an isosceles trapezium with three sides of length one unit and one side of length two units. Extend the two non-parallel sides of the trapezium to form a triangle as shown.



Lines PT and QS are parallel. Hence, using corresponding angles, we know that $\angle TPR = \angle SQR$ and also that $\angle PTR = \angle QSR$. This shows that the two triangles PRT and QRS have equal angles and so are similar. Therefore $\frac{x}{x+1} = \frac{1}{2}$ which has solution $x = 1$ and $\frac{y}{y+1} = \frac{1}{2}$ which has solution $y = 1$. Hence triangle QRS is equilateral and so its angles are all 60°. This means that the base angles of the trapezium are also both 60°.

Alternative solution:



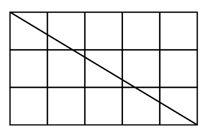
The only way we can have four positive integers that add up to 5 is $1 + 1 + 1 + 2 = 5$. So the trapezium must be as shown in the diagram on the left. Let this trapezium be $WXYZ$ as shown. Let U and V be the points where the perpendiculars from X and Y meet WZ. Since $WZ = 2$ and $UV = XY = 1$, we have $WU + VZ = 1$. Therefore, if we put together the two right-angled triangles XUW and YVZ, we obtain the equilateral triangle MZW shown on the right. As this is an equilateral triangle, $\angle XWZ = \angle YZW = 60°$.

10. B In any set of consecutive integers, there will either be the same number of odd and even numbers, one more odd number or one more even number. Hence the fraction of odd numbers in a set of consecutive integers will either be $\frac{1}{2}$ or be of the form $\frac{n}{2n-1}$ or of the form $\frac{n}{2n+1}$. The given percentages can be reduced to fractions as follows: $40\% = \frac{2}{5}$, $45\% = \frac{9}{20}$, $48\% = \frac{12}{25}$, $50\% = \frac{1}{2}$ and $60\% = \frac{3}{5}$. Of these, the only one not in an acceptable form is 45%.

11. A As the digits involved are 0, 1, 2 and 3, the largest difference will occur when the first digit changes. Hence the only cases that need considering are the change from 1320 to 2013 (difference 693) and the change from 2310 to 3012 (difference 702). This means the largest difference is 702.

12. A The value calculated for all four points will be negative. The least value will be obtained by calculating the most negative y-coordinate ÷ least positive x-coordinate. The most negative y-coordinates are at P and Q while the least positive x-coordinates are at P and S. Hence the point that will give the least value is P.

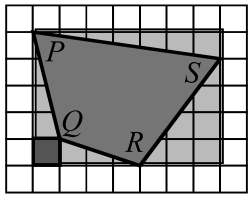
13. E The 6×10 grid can be divided into four 3×5 grids, each intersected by only one diagonal line as shown.



Each time the diagonal crosses a grid line, it enters a new cell.
From a start point in the top left corner of the grid, the line crosses two horizontal grid lines and four vertical grid lines to reach the bottom right corner. On the 3×5 grid, the line does not pass through any points at which the grid lines intersect. The number of cells in the 3×5 grid that the line intersects is $1 + 2 + 4 = 7$. Hence the total number of cells that are *not* intersected is $6 \times 10 - 4 \times 7 = 32$.

14. C Looking horizontally from behind, John will see the largest number of cubes in each column in the table. This means that, from his left, he will see 2, 3, 3 and 4 cubes. Therefore, the shape he will see is C.

15. B Surround the quadrilateral $PQRS$ by a rectangle with sides parallel to the grid lines as shown. The area of the rectangle is $14 \times 10 = 140\,\text{cm}^2$. The area of quadrilateral $PQRS$ can be calculated by subtracting from this the sum of the areas of the four triangles and one square that lie outside $PQRS$ but inside the rectangle from the area of the rectangle. This gives the area of $PQRS$ as



$$140 - \tfrac{1}{2} \times 14 \times 2 - \tfrac{1}{2} \times 8 \times 6 - \tfrac{1}{2} \times 6 \times 2 - 2 \times 2 - \tfrac{1}{2} \times 8 \times 2 = 140 - 14 - 24 - 6 - 4 - 8 = 84\ \text{cm}^2.$$

16. D The expression 2013^6 can also be written as $(2013^3)^2$. So $1^2, 2^2, \ldots, (2013^3)^2$ is the list of squares and hence $S = 2013^3$. Similarly $2013^6 = (2013^2)^3$ and so $Q = 2013^2$. Hence $S = 2013Q$.

17. D Adam must have removed the final digit of his number before adding or the final digit of the sum would have been an even number. If his original number was $ABCDE$ then, using this, we have $52713 = ABCDE + ABCD = 11 \times ABCD + E$. However $52713 \div 11 = 4792$ remainder 1 so Adam's original number was 47921 which has a digit sum of 23.

18. C The question states that the number of trees between any two maple trees must not equal three. Hence, in any block of eight trees, wherever a maple tree is placed, there must be a corresponding linden tree either four places in front of it or four places behind it. This means that the number of maple trees in any row of eight trees cannot exceed the number of linden trees. This means that in a row of 20 trees, no more than eight of the first 16 trees can be maples and so no more than 12 of the 20 can be maples. This can be achieved as shown below:

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19. B Andrew finished 21st so 20 runners finished in front of Andrew. This means that Dean finished ahead of $1\tfrac{1}{2} \times 20 = 30$ runners. Let x be the number of runners who finished ahead of Dean. This means that the number of runners who finished after Andrew was $2x$. By considering the total number of runners in the race in two different ways, we obtain the equation $x + 1 + 30 = 20 + 1 + 2x$. This has solution $x = 10$. Therefore, the number of runners in the race is $10 + 1 + 30 = 41$.

20. C In each net, the central 4×1 rectangle can be folded round to form the front, the sides and the back of a cube. The remaining triangles, if correctly positioned, will then fold to form the top and the bottom of the cube. To complete the cube, the triangles must fold down so that the shorter sides of each triangle are aligned with different edges of the top (or bottom) of the cube. It can be checked that this is the case in nets A, B, D and E. However, in net C, the two shorter sides of the lower two triangles would, when folded, align with the same two edges and so could not form a complete face of the cube.

21. A Label the cars 1, 2, 3 and 4 and their original junctions P, Q, R and S respectively. Whichever junction car 1 leaves by, any of the other three cars could leave by junction P. Once car 1 and the car leaving by junction P have been assigned their junctions, we have to consider the other two cars and the other two junctions. However, at least one of these remaining junctions will be the original junction of one of these two cars. Therefore there will be exactly one way in which the two remaining cars can leave by the two remaining junctions. So car 1 can leave by one of three junctions and, for each of these, the remaining cars can leave the roundabout in three different ways. Hence the total number of ways the cars can leave the roundabout is $3 \times 3 = 9$.

22. B From the definition of the sequence, it can be seen that the terms repeat in blocks of three. The sum of the first three terms is -1. In a sequence of 2013 terms, there will be $2013 \div 3 = 671$ sets of three terms. Hence, the sum of 2013 terms of the sequence is $671 \times (-1) = -671$.

23. D In the orderings, the only way it is possible for a smaller number to occur before a larger number would be if the pie corresponding to that larger number has not yet finished baking. In option D, 2 occurs before 3 but as pies 4, 5 and 6 have already been eaten, pie 3 would also have been baked. This means that option D is not possible. (It can easily be checked that all the other options do give possible orderings.)

24. B Let the values at the vertices P, Q, R and S be p, q, r and s respectively. The values on the edges are equal to the sum of the values at the vertices connected by that edge. Each vertex is at the end of three edges. Also, the sum of the values on all the edges and the values on all the vertices must be the same as the sum of the numbers 1 to 11 (excluding 10). Therefore $3(p+q+r+s)+p+q+r+s =$ $1+2+3+4+5+6+7+8+9+11$. This simplifies to $4(p+q+r+s) = 56$ or $p+q+r+s = 14$. Edge PQ is marked with 9 so that $p+q=9$. This leaves $r+s=5$ so edge RS will be marked with 5.

Alternative solution:

Labels 1 and 2 must be placed on vertices as it is impossible to find two numbers from the list that will add to give either 1 or 2. This implies that 3 must be placed on an edge between 1 and 2. Once 3 has been placed on an edge, 4 must be placed on a vertex as the only two numbers in the list that add to give 4 are 1 and 3 and 3 is not on a vertex. Then 11 must be placed on an edge as it is the largest number and so could not be part of any sum. Since the two largest vertex numbers must add to give the largest edge number, the remaining vertex has a value of 7. Hence the numbers on the vertices are 1, 2, 4 and 7. Edge PQ is marked with $9\,(= 2 + 7)$ so edge RS must be marked with $5\,(= 1 + 4)$.

25. C The three greatest divisors of N are the values obtained by dividing N by its three smallest divisors. To discover when it is possible to have the three greatest divisors of N adding to a value greater than N, it is only necessary to consider the cases when N is divisible by small integers. If N is divisible by 2, 3 and 4 then the sum of the three greatest divisors will be $\frac{N}{2} + \frac{N}{3} + \frac{N}{4} = \frac{13}{12}N > N$. Similarly, if N is divisible by 2, 3 and 5 then the sum of the three greatest divisors will be $\frac{N}{2} + \frac{N}{3} + \frac{N}{5} = \frac{31}{30}N > N$.

We then note that $N\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}\right) = N$ and that $N\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{5}\right) < N$ so that no further cases need to be considered. In both cases where the sum of the divisors is greater than N, N is divisible by 2 and 3. Hence all such N with the desired property are divisible by 6.