Mentoring Scheme

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Pythagoras

Sheet 1

Questions

This programme of the Mentoring Scheme is named after Pythagoras of Samos (c. 569–475 BCE).

See http://www-history.mcs.st-and.ac.uk/Biographies/Pythagoras.html for more information.

1. Write down the units digits of all the square numbers from 12 to at least 202.

How can you tell that 573 is not a square number?

2. What is the 2017th character in the sequence

$$ABCDEDCBABCDEDCBABCDEDCBAB$$

where the pattern $ABCDEDCB$ is repeated again and again?

3. A polygon of 100 sides has all possible diagonals drawn from a chosen vertex to the others.

How many triangles are formed? What is the sum of the interior angles of the 100 sided polygon?

[You should assume that all the interior angles are less than 180°. This is called a *convex* polygon.]

4. To make 'short' pastry one uses flour to fat (butter, margarine, etc.) in the ratio 2 : 1. To make 'flaky' pastry requires a ratio of 4 : 3.

I have 3 kg of flour and 2 kg of fat and wish to use all the ingredients making some of each type of pastry. How much flaky pastry do I make?

5. The exterior angles of a triangle are in the ratio 4 : 5 : 6. What is the least interior angle?

[The exterior angle of a polygon is the angle between one side and its neighbour extended.]

6. You tell your friend that there is the same chance of obtaining an even number of heads when tossing four coins as of a head showing when tossing one coin. S/he does not believe you. How do you convince your friend of the truth of your statement?

7. In a triangle ABC, the length of AB is 2 and the length of AC is 1. C is located so that a circle using BC as diameter will pass through A.

If you have not met this idea before, start by drawing the circle and then draw a rectangle inside it with ratio of sides 2 : 1. Draw the diagram carefully and reasonably large. Prove that $BC = \sqrt{5}$.

> This sheet is named after Pythagoras whose theorem you will need to use to solve this problem. What more can you find out about him and his followers? Was Pythagoras the first person who used this theorem or did earlier people know about it? A useful website about mathematicians and their discoveries is at: www-history.mcs.st-and.ac.uk

8. Construct a sequence of five different single digit positive integers a, b, c, d, e so that :

b divides into $a + b$ but a does not divide into $a + b$;

c divides into $a + b + c$ but a and b do not do so;

d divides into $a + b + c + d$ but a, b and c do not do so;

e divides into $a + b + c + d + e$ but a, b, c and d do not do so.