United Kingdom
Mathematics Trust

Mentoring Scheme

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Mary Cartwright

Sheet 1

Questions

This programme of the Mentoring Scheme is named after Dame Mary Lucy Cartwright (1900–1998).

See http://www-groups.dcs.st-and.ac.uk/history/Biographies/Cartwright.html for more information.

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1. This is the first of the Cartwright series of mentoring sheets, named after *Dame Mary Lucy Cartwright*, a mathematician who started the important mathematical field of *chaos theory*, proving some of the important results. What can you find out about her, and her areas of work?

2. Find all real numbers x such that $x - 3 = \sqrt{2x - 3}$.

3. Abby holds six pieces of string in her hand with the ends of each of the strings sticking out above and below her hand. Rhiannon ties the upper ends together in pairs, and then does the same with the lower ends. If she ties the pairs randomly, what is the probability that all six pieces of string will form a single loop?

4. Find all pairs of integers x and y such that $xy + 3x - 4y = 29$.

5. Suppose that

$$x + \frac{1}{x} = 5$$

(a) What is the value of

$$x^2 + \frac{1}{x^2}$$

(b) What is the value of

$$x^5 + \frac{1}{x^5}$$

6. (a) Prove that $a^2 + b^2 \geq 2ab$ for all real numbers a and b. When does equality occur?

(b) Prove that $a^2 + b^2 + c^2 \geq ab + bc + ca$ for all real numbers a, b and c. When does equality occur?

7. Find all integers n such that $n^2 - 7n + 10$ is divisible by $n - 3$.

8. Is it possible to place positive integers at each of the vertices of the cube in such a way that for each pair of numbers connected by an edge one is divisible by the other, and no other pair of numbers have this property. *(i.e. no two unconnected vertices have one number divisible by the other.)*

9. Solve the equation $x^4 - x^3 - 4x^2 - x + 1 = 0$.

Hints

1. http://www-groups.dcs.st-and.ac.uk/history/Biographies/Cartwright.html might be a good place to start.

3. What happens after all the top ends have been joined together?
What is the probability that the first knot below does not produce a loop? What about the second knot?

4. Can you adjust the left hand side of the equation such that it then factorises?

5. You don't need to calculate the value of x to solve this question.
What happens when you calculate successive powers of $x + \frac{1}{x}$?

7. Can you find an expression containing n^2 that is divisible by $n - 3$? What is left over?

8. How might prime numbers be useful in doing this? You might find it helpful to first of all try the same question with a cube replaced by a square.

9. How can you use the idea from question 5 to help you?