United Kingdom
Mathematics Trust

Mentoring Scheme

Supported by OxFORD ASSET MANAGEMENT

Hypatia

Sheet 1

Questions

This programme of the Mentoring Scheme is named after Hypatia of Alexandria (c. 370–415 CE).

See http://www-history.mcs.st-and.ac.uk/Biographies/Hypatia.html for more information.

Enquiries about the Mentoring Scheme should be sent to:

*Mentoring Scheme, UK Mathematics Trust, School of Mathematics,
University of Leeds, Leeds LS2 9JT*

☎ 0113 343 2339 0113 343 5500 mentoring@ukmt.org.uk www.ukmt.org.uk

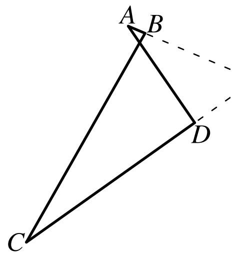
Before you start the questions, see what you can find out about Hypatia, one of the few women to influence mathematics in the Ancient Greek world.

1. A prime number p is chosen so that $2011 + p$ is a power of 2.
What is the minimum possible value of $27p$?

2. Work out the day of the week for 14th September 1752, the first day of the new calendar in England when an Act of Parliament brought the calendar into alignment with the Gregorian calendar used in most of Western Europe by that time. You need to know that leap years occur every 4 years with the exception that every 100th year is *not* a leap year. There is even an exception to that! Every 400th year *is* a leap year.

3. You are given that n is a positive integer with the property that when we add n and the sum of its digits, we obtain the number 313. What are the possible values of n?

4. In the diagram AD and BC intersect at X. $\angle ABX = 90°$. $\triangle ABX$ and $\triangle CDX$ are similar.
$AB = 4$, $AX = 5$ and $AD = 20$.
AB and CD extended intersect at Y but Y is not shown on the diagram.
What is the area of $BXDY$?



5. A sequence of numbers of which the 1st is -5 and the 26th is $+2$ is defined by the rule that each number is the sum of the one before it and the one after it. Find the 12th number.

6. Let a and b be positive integers such that $b = \sqrt{a\sqrt{a\sqrt{a}}}$

and such that $a > 1$. Find the least value of $a + b$.

7. Show that $1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \left(\frac{1}{9} + \frac{1}{10} + \cdots + \frac{1}{15} + \frac{1}{16}\right) > 3$
What happens to the sum of the fractions on the left as the number of terms increases?

8. How many pairs of distinct numbers can you choose from the set $\{1, 2, 3, 4, \ldots, 2016, 2017\}$ such that their sum is an even number?

---

2. You may have completed question 6 on Pythagoras sheet 2.
4. You will benefit from drawing an accurate diagram which includes Y.
5. If you let the first two numbers be x and y, then how much more of the sequence can you work out without having to introduce a third unknown?
6. Square the equation on both sides more than once.
7. The brackets are inserted deliberately to help you.
8. You may like to look back to Pythagoras sheet 2 question 5 to see if you can spot link.

