United Kingdom
Mathematics Trust

Mentoring Scheme

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Archimedes

Sheet 1

Questions

This programme of the Mentoring Scheme is named after Archimedes of Syracuse (287–212 BCE).

See http://www-history.mcs.st-and.ac.uk/Biographies/Archimedes.html for more information.

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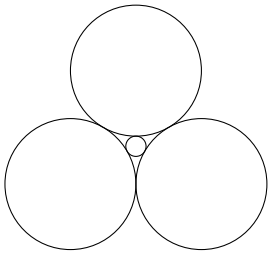
Find out what you can about Archimedes. Do any of his original manuscripts still exist?

When you are unfamiliar with a topic, you may find it useful to refer to the UKMT notes *which are suited to the earlier levels of the mentoring scheme.*

1. Three circles of radius 1 unit touch each other externally as shown in the diagram. A small circle is placed in the middle so that it touches each of these circles. What is its radius?



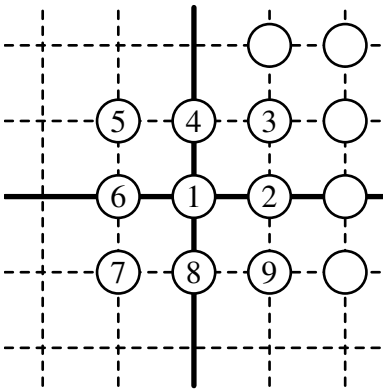
2. You are given that $2 + y = x^2$. Put $x = 2 + a$ and $y = 2 + b$ where both a and b are between 0 and 1. Show that if y is close to 2, then x is closer.
What about the case where x is a little less than 2?

3. Is it possible to draw a triangle in which the sum of any two angles is less than $120°$? You must prove your answer.

4. We choose six (different) numbers out of the ten integers $1, 2, 3, \ldots, 10$. Prove that their product is divisible by a perfect square greater than 1.

5. At the reading of a will, Prudence and Ravi are told that they will inherit one sixth of the total money between them. There are conditions. They are not allowed to inherit the same amount as each other but each amount must be a simple fraction of the money, that is a fraction with numerator 1. If Prudence inherits $\frac{1}{x}$ and Ravi inherits $\frac{1}{y}$ of the total money, show that $xy - 6x - 6y = 0$.
In how many ways can the money be split between them?

6. The positive integers starting at 1 are written down on a graph at the points $(0,0), (1,0), (1,1), (0,1), (-1,1), (-1,0)$ forming an anti-clockwise spiral of integer grid points around $(0,0)$ as shown in the diagram. Find the coordinates of the point where 2018 is written.



7. A triangle has two vertices $A(-3,1)$ and $B(3,-1)$. The third vertex C is at $(2,k)$. Find all the values of k such that the triangle is isosceles.

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1. Refer to the UKMT notes on the centroid of a triangle and on surds.
2. It is instructive to study a graph of $y = x^2 - 2$ and look at how x and y change near the point $(2,2)$.
3. What happens if you assume all pairs of angles add to less that $120°$?
4. What is the maximum number of integers you can choose so that the product can not be divided by a square > 1?
5. Think about setting up an equation and multiply out so that you have no fractions. Show that your equation can be rearranged to look like $(x-a)(y-a) = b$ by choosing the values of a and b appropriately.
6. Look at the numbers at the bottom right corner of the spiral each time a square is completed.
7. You are advised to draw an accurate diagram on graph paper to analyse this problem. Which sides of $\triangle ABC$ could be the base of the isosceles triangle?