BRITISH MATHEMATICAL OLYMPIAD

PREPARATION SHEET

INTRODUCTION

There is no set syllabus for the BMO papers. **The questions are designed to be non-standard, requiring ingenuity in application of mathematical ideas and techniques**. On the other hand, the questions do fall into a few distinct categories and there are some basic techniques which it is useful to know. The idea of this sheet is to outline these for everyone's benefit, but more importantly to make clear the BMO philosophy and what is expected in terms of full solutions. The work expected and the way it is rewarded are fundamentally different from GCSE and A-level. **Candidates should NOT be discouraged if marks are low. It is the experience, the challenge and the opportunity to think protractedly about a problem which are important**.

It should be clearly understood that all solutions require rigorous justification (proof) and not just conjectures based on pattern-spotting. It is perfectly possible that candidates may get say 1/10 for a question for which they think they will get full marks! The most likely reason is that they have not understood that they have to **PROVE** their assertions - of course proofs can take many different styles, and as long as an argument is valid and complete it will gain full marks, even if it is inelegant. A knowledge of proof by induction is highly desirable. It is also important to be **systematic** in listing possibilities and to make such systematic enumeration **explicit**. It is important to understand the meaning of the phrase "if and only if" and also be familiar with factorial notation and terms like arithmetic progression, rational numbers, natural numbers and similar descriptions.

Candidates should have the confidence to **THINK** and try different approaches. There is a temptation to believe that there is some formula which they should know but don't, which might lead some to abandon a question without giving themselves a chance. Whilst this may be the case, it is more likely that if they took time to try things out and get into the problem, they might come up with a method which might work.

GEOMETRY

For BMO1, nothing much is expected beyond the circle theorems done for GCSE including the Alternate Segment Theorem. It is useful to be able to identify cyclic quadrilaterals and then use circle properties, even when circles are not mentioned in the question. Most questions, certainly for BMO2 will require some construction, so some imagination is needed. Also for BMO2 it is helpful if candidates know the four main centres of a triangle - the circumcentre, orthocentre, incentre and centroid and basic results concerning these, and also Heron's formula for the area of a triangle.

TRIGONOMETRY

Knowledge of the Cosine Rule and the FULL Sine Rule (including 2R) is assumed. Obviously the more fluent candidates are with double angle formulae etc, the more weapons they have in their armoury, but questions relying on these would be unusual and probably susceptible to other methods.

FUNCTIONAL EQUATIONS

Candidates often seem to be fazed by such questions. They need to have some idea of substituting in values or expressions into these fairly abstract equations in order to find out things about the functions. It is probably simply a question of trying one or two before sitting the BMO papers.

ALGEBRA

Understanding of quadratics, the Factor Theorem and its use in factorising say $x^3 - y^3$ are assumed. The only inequality (apart from the fact that all squares are non-negative) which is assumed for BMO1 is the AM-GM inequality, that for any set of positive numbers the Arithmetic Mean ≥ Geometric Mean, with equality when all the values are equal. For BMO2, a knowledge of the Cauchy-Schwarz Inequality might be useful, which states that for all real numbers $a_1, ..., a_n$, $b_1, ..., b_n$, $(a_1b_1 + ... + a_nb_n)^2 \leq (a_1^2 + ... + a_n^2)(b_1^2 + ... + b_n^2)$.

NUMBER THEORY

At BMO level, most questions in this area will involve finding integer solutions to equations (i.e. Diophantine Equations) for which an understanding of factorisation and the significance of primes is essential. A common situation is to find integer solutions of, say, $xy + x + y = 2004$ or similar which relies on realising that $xy + x + y + 1$ can be factorised to $(x + 1)(y + 1)$. Some knowledge of modular arithmetic may be particularly helpful at BMO2 level. BMO1 often uses (implicitly) arithmetic modulo 10, so the idea of extending this is quite important. Number bases, rules for divisibility and the idea of parity are all helpful. Fermat's Little Theorem could be useful for BMO2.

COMBINATORICS

For BMO1 a knowledge of Binomial Coefficients should be sufficient. It might be useful to know the Pigeon-hole Principle (basically that you can't fit pigeons into boxes without some sharing if there are more pigeons than boxes) particularly for BMO2. Most questions in this area will just rely on these few ideas. It is also helpful to have some idea of recurrence relations in building up a method of counting. Another useful idea is to represent situations using vertices and edges as in Graph Theory.

HELPFUL TEXTS AND RESOURCES

An extremely helpful book for anyone preparing for BMO papers is *A Mathematical Olympiad Primer* by Dr Geoff Smith (£11). Booklets containing past BMO1 papers and their solutions are available for £2.50 per volume. Both can be purchased via the UKMT website at **www.ukmt.org.uk**
Additionally, *The Mathematical Olympiad Handbook* by A. Gardiner (OUP) ISBN 0-19-850105-6 is highly recommended.
Past papers (no solutions) can be downloaded from the BMO website at **www.bmoc.maths.org**

Web sites

BMO	www.bmoc.maths.org
NRICH	www.nrich.maths.org.uk
Maths Digest	www.mth.uct.ac.za/digest/index.html
Hungarian problems	www.komal.hu/info/bemutatkozas.e.shtml