

BMOS Mentoring Scheme

Intermediate Level 2013-14

Sheet 5

These questions are not necessarily in order of difficulty, and you do not have to attempt them in order.

1. I have thought of five numbers. If I add three of them in each possible way, I get 10, 14, 15, 16, 17, 17, 18, 21, 22 and 24. What are my numbers?
2. My rectangular garden is a metres long and b metres wide. I have made a plan of it, to the scale 1:10. I place the plan flat on the ground somewhere in my garden, so that the whole map is entirely within the garden, is in the right direction and is the right way up. Is there a point in the garden that is directly underneath the point representing it on the map? Is there more than one such point?
3. The two-digit number X is equal to four times the product of the digits of the two-digit number Y . Similarly, Y is equal to four times the product of the digits of X . Find all possible such pairs X, Y .
4. Find, in terms of the positive real numbers p and q , the area of the quadrilateral whose vertices have coordinates (p, p) , (q, p) , $(2q, q)$ and $(p + q, q)$.
5. How many years between 2001 and 2100 inclusive have the property that dividing the year number by each of 2, 3, 4 and 5 leaves a remainder of 1?
6. Eight snooker players have entered a tournament. It will be a knockout tournament, with three rounds. In how many different ways can I set up the tournament? The order of the two players in each match doesn't matter. For example, Gavin vs. Jo would be the same as Jo vs. Gavin. The order of the matches is not important either: if Gavin plays Jo, then Paul plays Jenny and the two winners meet in the next round, this is the same as if Jenny plays Paul and then Gavin plays Jo and the two winners meet in the next round.
7. Let x, y and z be positive real numbers such that $xyz = 1$. Show that

$$\frac{1}{1+x+xy} + \frac{1}{1+y+yz} + \frac{1}{1+z+zx} = 1.$$

8. Ten points are placed inside a unit square (a square with sides of length 1). Show that there is a pair of points at most $\frac{\sqrt{2}}{3}$ apart.