BMOS Mentoring Scheme

Intermediate Level 2013-14

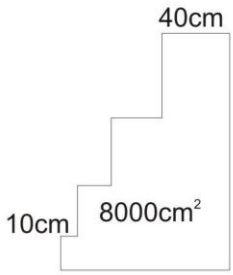
Sheet 2

These questions are not necessarily in order of difficulty, and you do not have to attempt them in order.

1. Use Euclid's Algorithm to find the highest common factor of 30073 and 83143. (Look at the solutions to the October sheet if you don't know about Euclid's Algorithm.)

   If you have time, try this question using prime factorisations. Which method is easier?

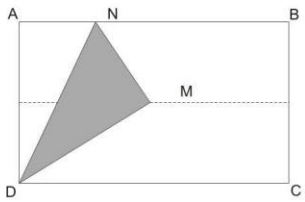
2. I have built a tombola stall as shown in the diagram.



   Each step is 10cm wider than the next step down, and each step is 10cm higher than the next step down. The lowest step is 10cm wide. If the area of the side is 8000cm^2, what is the height of the lowest step?

3. Find all real numbers x such that $\sqrt{-3+4x} - \sqrt{13-4x} = 2$.

4. A rectangular piece of paper has a line drawn down the middle. One corner is then folded along DN so that the corner A coincides with a point M on the mid-line, as shown. Prove that $\angle ADN = 30°$.



5. How many four-digit numbers have precisely three different digits, such as 2005?

6. Show that, for all real numbers x, y, z, we have $x(x-y) + y(y-z) + z(z-x) \geq 0$. When does equality occur?

7. Find all positive integers (whole numbers) n such that $27n + 37$ is divisible by $3n + 1$.

8. I have to select four of the seven dwarves to play a game of bridge. In how many ways could this be done? (The order in which the dwarves are chosen does not matter.)

   Can you find a formula for the number of ways in which r dwarves can be chosen from a group of n dwarves?