

BMOS Mentoring Scheme

Intermediate Level 2013-14

Sheet 1

These questions are not necessarily in order of difficulty, and you do not have to attempt them in order.

1. I have 10 pairs of socks in my drawer, but all of the socks are separate. Each pair is a different colour. I shut my eyes and pull out some socks. How many must I take out to **guarantee** that I get a pair? How many more do I need in order to get a second pair?
2. Find the lowest common multiple and highest common factor of 47880 and 67620.
3. $ABCD$ is a rectangle. Sides AB and CD have length 10cm, and sides BC and DA have length 6cm. A point E is marked on side CD such that it is 2cm from C . Show that $\angle AEB = \angle BEC$.
($\angle ABC$ means the angle between AB and BC , for example.)
4. 12 has 6 factors: 1, 2, 3, 4, 6, 12. Find a criterion for a number to have an odd number of factors.
(That is, find some simple property that is held by all numbers with an odd number of factors, but is not held by any number with an even number of factors.)
5. Find all real numbers x such that $\sqrt{13 - 4x} = 2x + 1$.
6. (i) For $a, b \geq 0$, prove that $\frac{a+b}{2} \geq \sqrt{ab}$. When can equality occur?
($\frac{a+b}{2}$ is the arithmetic mean of a and b ; \sqrt{ab} is the geometric mean.)
(ii) For $a, b \geq 0$, prove that $\sqrt{ab} \geq 1 / \left(\frac{1}{2} \left(\frac{1}{a} + \frac{1}{b}\right)\right)$. When can equality occur?
($1 / \left(\frac{1}{2} \left(\frac{1}{a} + \frac{1}{b}\right)\right)$ is the harmonic mean of a and b .)
7. How many ways are there of arranging the letters abc ? What about $abcd$?
(abc is different from bac , for example.)
How many ways are there of arranging the letters *camera*? What about *banana*?
8. The *incircle* of a triangle is the circle inside the triangle that is tangent to all three sides (it just touches each side).
Let ABC be a triangle with side lengths a, b, c and area $[ABC]$. Let r be the radius of the incircle of the triangle. Show that $[ABC] = \frac{1}{2}r(a + b + c)$.