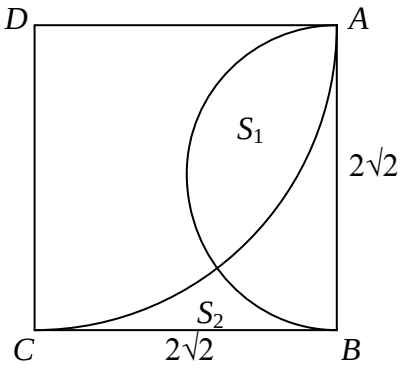
UKMT MENTORING SCHEME (Intermediate Level)
April 2013 (Sheet 7)
Questions

1. The square shown has side $2\sqrt{2}$. A semicircle diameter AB is shown, and also a quadrant centre D, passing through A and C. The areas of the two regions shown are S_1 and S_2. Calculate $S_1 - S_2$.



2. Prove that among any 18 consecutive three-digit numbers there must be one which is divisible by the sum of its digits.

3. In a trapezium $PQRS$, PQ and RS are perpendicular to QR, $PQ = 8$, $QR = 10$ and $RS = 3$. T is a point on QR such that $PTS = 90^o$. Find all possible values for the area of ΔPTS.

4. Find all solution pairs (n, m) of $2^n + 7 = m^2$ in which n and m are both integers.
[*You must prove there are no other solutions.*]

5. In an urn, there are only black and white marbles, the total number of which, rounded to the nearest hundred, is 1000. The probability of pulling out two black marbles is $^{17}/_{43}$ greater than the probability of pulling out two white marbles. How many of each type of marble are there in the urn?

6. Every member of a given sequence, beginning with the second, is equal to the sum of the preceding one and the sum of its digits. The first member is 1. Is there, among the members of this sequence, a number equal to 123456?

7. Inside a given rectangle is inscribed a quadrilateral, which has a vertex on each side of the rectangle. Prove that the perimeter of the inscribed quadrilateral is at least double the length of a diagonal of the rectangle.

8. Theo has four children. The age in years of each child is a positive integer between 2 and 16 inclusive, and all their ages are different. A year ago the square of the age of the oldest child was equal to the sum of the squares of the ages of the other three. In one year's time, the sum of the ages of the oldest and youngest will be equal to the sum of the squares of the other two. Find all possibilities for their ages.

Deadline for receipt of solutions: 30th April 2013