UKMT MENTORING SCHEME (Intermediate Level)
February 2013 (Sheet 5)
Questions

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1. Find the smallest positive integer that appears in all the arithmetic sequences:

$$22,\ 33,\ 44,\ 55,\ \ldots$$
$$24,\ 37,\ 50,\ 63,\ \ldots$$
$$25,\ 39,\ 53,\ 67,\ \ldots$$

2. Find the number of digits in $5^{20} \times 4^{13}$ when it is worked out and written as a single number.

3. An isosceles right-angled triangle is inscribed inside a circle of radius 1. Find the radius of the incircle of the triangle.
[*The incircle of a triangle is the circle which touches all three sides of the triangle.*]

4. If $x + y = 5$ and $xy = 2$, find the values of $x^3 + y^3$ and $x^4 + y^4$.

5. Four dice, coloured red, blue, green and black are rolled. In how many ways can the product of the numbers rolled be 36 (where, for example, a red 4 is considered different to a blue 4)?

6. From the set of consecutive integers $\{1, 2, 3, \ldots n\}$, five integers that form an arithmetic sequence are deleted. The sum of the remaining integers is 5000. Determine all values of n for which this is possible and for each acceptable value of n, determine the number of five-integer sequences possible.
[*An arithmetic sequence is a sequence where consecutive terms differ by a constant amount.*]

7. Determine (without a calculator!) the value of $\dfrac{1^4 + 2012^4 + 2013^4}{1^2 + 2012^2 + 2013^2}$.

8. Determine the least natural number n for which the following result holds:
No matter how the elements of the set $\{1, 2, \ldots, n\}$ are coloured red or blue, there are integers x, y, z, w in the set (not necessarily distinct) of the same colour such that $x + y + z = w$.

Deadline for receipt of solutions: 28th February 2013