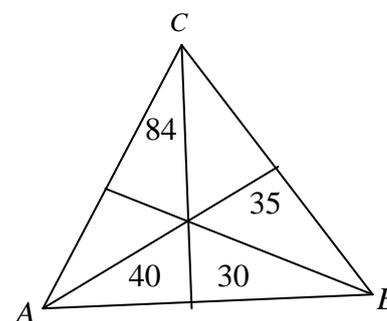


Here is the third sheet for this year. I hope you enjoy tackling the problems.

1. As shown in the figure, triangle  $ABC$  is divided into six smaller triangles by lines drawn from the vertices through a common interior point. The areas of four of these triangles are as indicated. Find the area of triangle  $ABC$ .



2. If  $x$ ,  $y$  and  $z$  are positive numbers satisfying  $x + \frac{1}{y} = 4$ ,  $y + \frac{1}{z} = 1$  and  $z + \frac{1}{x} = \frac{7}{3}$ , find the value of  $xyz$ .

3. The expression  $\binom{n}{r}$  means  $\frac{n!}{r!(n-r)!}$  where  $n! = n \times (n-1) \times (n-2) \times \dots \times 1$ .

Find the largest two-digit prime number which is a factor of  $\binom{200}{100}$ .

4. The numbers 1447, 1005, and 1231 are all four-digit numbers beginning with 1 which have exactly two identical digits. How many such numbers are there?

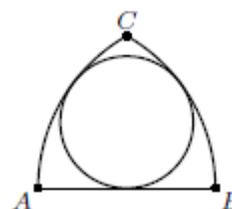
5. Find four positive integers  $a$ ,  $b$ ,  $c$  and  $d$  which have a product of  $8!$  and satisfy

$$ab + a + b = 524$$

$$bc + b + c = 146$$

$$cd + c + d = 104$$

6. The circular arcs  $AC$  and  $BC$ , as shown, have centres at  $B$  and  $A$  respectively. If the length of the arc  $BC$  is 12, find the circumference of the circle which is tangential to the line  $AB$  and also to the arcs  $AC$  and  $BC$ .



7. Eight congruent equilateral triangles, each of a different colour, are used to construct a regular octahedron. How many distinguishable ways are there to construct the octahedron? (Two coloured octahedra are distinguishable if neither can be rotated to look like the other.)

8. For  $\{1, 2, 3, \dots, n\}$  and each of its non-empty subsets a unique *alternating sum* is defined as follows: Arrange the numbers in the subset in decreasing order and then, beginning with the largest, alternately add and subtract successive numbers. (For example, the alternating sum for  $\{1, 2, 4, 6, 9\}$  is  $9 - 6 + 4 - 2 - 1 = 6$  and for  $\{5\}$  it is simply 5.) Find the sum of all such alternating sums for  $n = 7$ .