UKMT MENTORING SCHEME (Intermediate Level)
November 2012 (Sheet 2)
Questions

Here is the second sheet of problems for this year. The questions are graded so that earlier questions tend to be easier. These questions are designed to be challenging, especially the later ones, so you should be prepared to think about them for a while and try things out (jot things down) as you begin to see what the question is about and how you might tackle it. They are not impossible!! They have been chosen to give you a good challenge and hopefully you will have the satisfaction of solving several of them. When you know how to do a problem, then try to write up your solution so that it is clear and concise, but also thorough. For example in Q5, you must show that you have ***all*** *the possibilities, not just find one that works. Q1 does not, on the other hand, require any explanation. Just do it!*

1. Put the digits 1 to 9 (once only each) into the frame so that the product of each three digits is as shown against each row and column.

	6	192	315
20			
126			
144			

2. *ABCD* is a square with side length 1. *P* is the midpoint of *CD* and *Q* is the intersection of *AP* and *BD*. Find the area of quadrilateral *BCPQ*.

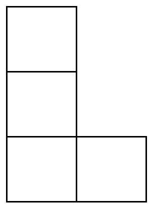
3. We define the *reverse* of an integer to be the number obtained by writing its digits in the opposite order. e.g. The reverse of 153 is 351.
The two-digit number 72 has the property that multiplying it by $^3/_8$ reverses it, since $^3/_8 \times 72 = 27$. Find all *three-digit* numbers and also all *four-digit* numbers which are reversed by multiplying them by $^3/_8$.

4. A straight line is drawn through the point *P* (3, 4). The line cuts the axes at *A* and *B* such that the area of triangle *OAB* is 24, where *O* is the origin. Determine the *x* – intercepts of all such lines.

5. Find all quadruples of integers (*a*, *b*, *c*, *d*) such that $52^a \times 77^b \times 88^c \times 91^d = 2002$.

6. Is it possible to cover an $n \times n$ chessboard with its centre square cut out with tiles shown in the figure opposite if
   a) $n = 5$ b) $n = 2011$?

   [*The tiles may be slid into any orientation, but not turned upside down.*]



7. Find all possible integer values of $\dfrac{m^2 + n^2}{mn}$ where m and n are integers.

8. Find all possible sets of three consecutive positive integers such that each of the three integers is either a prime or a power of a prime.

Deadline for receipt of solutions: 30th November 2012