UKMT MENTORING SCHEME (Intermediate Level)
March 2012 (Sheet 6)
Questions

1. The number m is the first of five consecutive integers that are respectively multiples of 3, 4, 5, 6 and 7. If $m > 7$, find the smallest possible value of m.

2. Two cylinders have diameters of 12 cm and 4 cm respectively. Find the length of the shortest band which can be fixed round the cylinders to hold them together.

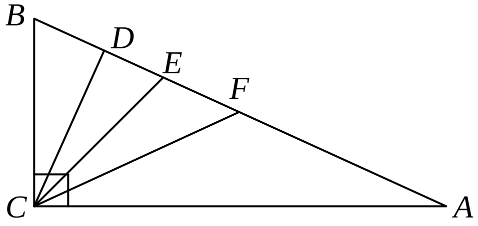
3. The sequence 9, 17, 24, 36 is one in which each term is the sum of the corresponding terms of two other sequences. The first of these is formed after the first two terms by each term being the sum of the previous two terms. The second is an arithmetic sequence (whose consecutive terms have a constant difference). Find the 7th term of the resulting sequence.

4. Find the smallest integer which has precisely 2012 positive integer factors.

5. Find the set of 100 consecutive odd positive integers whose sum is 100^{100}.

6. Show that given any 16 composite integers less than 2500, at least two of them will have a prime factor in common.

7. In the right-angled triangle ABC (as shown), CF is the median drawn to the hypotenuse AB, CE is the bisector of the angle ACB, and CD is the altitude to AB (i.e. CD is perpendicular to AB.) Prove that $\angle DCE = \angle ECF$.



8. Let x, y be positive real numbers such that $10x + 3y = 35$.
Find the maximum possible value of x^4y^3.
[*Hint: recall that on last month's sheet we proved (in outline) the "Arithmetic Mean - Geometric Mean" Inequality, which says that for any set of positive numbers* $a_1, a_2, ..., a_n$, *it is true that* $\frac{a_1 + a_2 + ... + a_n}{n} \geq \sqrt[n]{a_1 a_2 ... a_n}$, *with equality when* $a_1 = a_2 = ... = a_n$.]

Deadline for receipt of solutions: 30th March 2012

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