UKMT MENTORING SCHEME (Intermediate Level)
October 2011 (Sheet 1)
Questions

Welcome to the 2011 Intermediate Mentoring Scheme. Each month you will be set a sheet of eight questions. Mostly the problems will respond to ingenuity rather than special methods, though gradually over the course of the year some methods will be introduced. A really good problem should make you think "How on Earth do I do that?" And then after you have thought about it for a while and tried a few things, then a method of solution might gradually occur to you, so don't give up too easily! On the whole the earlier questions tend to be easier and the last couple will often be quite difficult. Ideally you should send your mentor or teacher ***full written solutions*** *by the deadline date.*

1. Find the sum of the digits in the square of the number 111 111 111.

2. A train takes ¼ minute to pass a telegraph pole, and ¾ minute to pass through a tunnel 540 metres long. What is the length of the train?

3. In ΔABC, the length of AB is 13cm, the length of AC is 15cm, and the length of the perpendicular from A to BC (i.e. the "altitude" from A) is 12cm. Find the two possible lengths of BC.

4. Circle K has diameter AB. Circle L touches K internally and also touches the line AB at the centre of circle K. Circle M touches L externally and K internally and also has tangent AB. Find the ratio of the area of circle K to the area of circle M.

5. How many odd numbers greater than 60000 can be made from the digits 5, 6, 7, 8, 9, 0 if no number contains any digit more than once?

6. The total area of all the faces of a cuboid is 22 cm^2, and the total length of all its edges is 24 cm. Find the length of any one of its internal diagonals.

7. Let n be an integer greater than 6. Prove that if $n - 1$ and $n + 1$ are both prime, then $n^2(n^2 + 16)$ is divisible by 720. Is the converse true?

8. Let G be a convex quadrilateral. Show that there is a point X in the plane of G with the property that every straight line through X divides G into two regions of equal area if and only if G is a parallelogram.

Deadline for receipt of solutions: 29th October 2011